

Interaction between Planetary-Scale Diffluent Flow and Synoptic-Scale Waves During the Life Cycle of Blocking

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ABSTRACT

In this paper, a new transient forced quasi-resonant triad interaction theory in a beta channel is proposed to investigate the interaction between planetary-scale diffluent flow composed of zonal wavenumbers 1–3 and synoptic-scale waves produced continuously by a synoptic-scale vorticity source fixed upstream of an incipient blocking region during the life cycle of blocking. It is shown that the superposition of initial three Rossby waves for zonal wavenumbers 1 (monopole), 2 (dipole), and 3 (monopole), which permit triad quasi-resonance, can represent an incipient blocking event. The synoptic-scale eddies may act to amplify the incipient blocking and to excite a blocking circulation with a strong meander, whose flow pattern depends on the initial amplitudes of the planetary waves and both the intensity and location of preexisting synoptic-scale waves. The onset (decay) of the planetary-scale split-flow blocking is mainly represented by a strong increase (decrease) in the amplitude of the zonal wavenumber 2 component, having a dipole meridional structure related to the preexisting synoptic-scale eddies. The typical persistence time of the model blocking was of about 20 days, consistent with observations of blocking patterns.

In our model, isolated asymmetric dipole blocking is formed by synoptic-scale waves. The instantaneous fields of total streamfunctions exhibit a remarkable resemblance to the synoptic maps observed during the life cycle of blocking. During the onset stage, the synoptic-scale waves are enhanced and split into two branches around the blocking region due to the feedback of the amplified blocking, in agreement with the observed changes of synoptic-scale waves in real blocking events. In addition, a diagnostic case study of blocking is presented to confirm the forced quasi-resonant triad interaction theory proposed here.

Key words: quasi-resonant triad interaction, blocking waves, synoptic-scale waves, blocking case

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1. Introduction

Observational, diagnostic, and numerical studies have supported the view that the nonlinear interaction of intense baroclinic cyclone-scale waves with barotropic, ultra-long waves plays a crucial role in forcing and sustaining blocking (Berggren et al., 1949; Rex, 1950a, b; Green, 1977; Frederiksen, 1982; Hansen and Chen, 1982; Hansen and Sutera, 1984; Ji and Tibaldi, 1983; Shutts, 1983, 1986; Illari, 1984; Colucci,

1985, 1987; Metz, 1986; Holopainen and Fortelius, 1987; Haines and Marshall, 1987; Malanotte-Rizzoli and Malguzzi, 1987; Vautard and Legras, 1988; Vautard et al., 1988; Tsou and Smith, 1990; Tanaka, 1991; Chen and Juang, 1992; Nakamura and Wallace, 1990, 1993; Nakamura et al., 1997; Ek and Swaters, 1994; Lupo and Smith, 1995a, b, 1998; Lupo and Bosart, 1999; Colucci, 2001). In this interaction process, intense upstream cyclogenesis precedes the growth of blocking downstream, and incipient block-

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ing that exists downstream of the cyclogenesis is a pre-requisite condition for the establishment of a blocking flow. The incipient blocking can be developed into a blocking flow by spatially and temporally persistent transport of potential vorticity (PV) associated with synoptic-scale waves (eddies) upstream (Hansen and Chen, 1982; Shutts, 1986; Colucci, 1985, 1987).

A dynamic link between a blocking dipole and the eddy forcing arising from the synoptic eddies has been established by Malguzzi (1993) analytically, who noted that the eddy forcing determines the steepening of dipole blocking. Unfortunately, he did not provide an explanation for why the real synoptic eddies tend to split as they enter the blocking region and also did not present the instantaneous change of a strong blocking circulation induced by the eddy forcing (Nakamura and Wallace, 1990). More recently, Luo (2000) has clarified the situation by establishing an eddy-forced envelope Rossby soliton theory whereby the onset and decay of a blocking flow associated with synoptic-scale eddies can be represented by the transfer between the level of dispersion and non-dispersion of the transient forced envelope Rossby soliton. Several studies on this aspect have been carried out (Luo, 2005a, 2005b, 2005c, 2005d). However, Colucci et al. (1981) showed that the split-flow blocking could be explained by free wave resonant interaction theory (Loesch, 1974). In fact, it is rather difficult for the planetary-scale waves to attain as high of amplitude as that of the blocking flow only through the resonant interaction among free triad planetary-scale waves. In this resonant interaction process, one planetary-scale wave grows in amplitude through the gain of energy from the other two. But the amplitude of the growing planetary wave is too small to create a blocking flow even in the strongest stage. If intense synoptic-scale waves are considered and are located upstream of an incipient blocking composed of preexisting triad planetary-scale waves of zonal wavenumbers 1–3, the fastest growing planetary-scale wave may attain the high amplitude required by real blocking flow. However, since eddy forcing is not involved in the resonant interaction model proposed by Loesch (1974) and further used by Colucci et al. (1981), this triad resonant interaction theory fails to explain the life cycle of observed blocking associated with synoptic eddies (Berggren et al., 1949, their Figs. 14–18 and Fig. 26). For this reason, Colucci et al. (1981) suggested that the interaction between cyclone-scale waves and triad resonant interacting planetary-scale waves might very well be relevant to observed blocking. However, apart from the quasi-resonance condition, the resonance condition cannot be strictly satisfied for the three planetary waves for zonal wavenumbers 1–3 in a beta-channel.

This motivates us to further investigate the interaction of quasi-resonant triad planetary-scale waves with synoptic-scale waves during the life cycle of blocking in an equivalent barotropic model.

Observational studies by Colucci et al. (1981) and Hansen and Chen (1982) have indicated that observed blocking flow is dominated by zonal wavenumbers 1–3. It can be shown that if these planetary waves have a monopole meridional structure, then both the resonance and quasi-resonance cannot be allowed in a beta-channel model. But three planetary-scale Rossby waves for zonal wavenumbers 1 (monopole), 2 (dipole) and 3 (monopole) allow triad quasi-resonance in some parameter ranges. The superposition of the quasi-resonant triad waves having different initial amplitudes can represent different incipient blockings. On this basis, the interaction of the different incipient blockings with synoptic eddies can be examined. In this paper, we will focus on the study of the interaction between an incipient blocking comprised of quasi-resonant triad planetary-scale waves and synoptic-scale waves, and will examine how both blocking circulation and synoptic eddies change instantaneously during the life cycle of a blocking (Berggren et al., 1949, their Figs. 14–18 and Fig. 26). Such a study will help us understand the physical mechanisms of the onset, maintenance, and decay of blocking by synoptic-scale eddies.

In the real atmosphere, the block-eddy interaction process is a baroclinic process (Frederiksen, 1982). However, including baroclinic process will complicate our problem even though it may be important in the early stage of blocking onset. In this paper, to emphasize the net contribution of barotropic synoptic eddies to blocking onset, an equivalent barotropic model will be used similar to the studies of McWilliams (1980), Shutts (1983), and Malguzzi (1993). The use of such a highly idealized model will simplify our problem considerably.

The purpose of the present paper is to propose a new transient forced quasi-resonant interaction theory, which is an extension of Colucci et al. (1981), seeking to clarify the interaction between planetary- and synoptic-scale waves during the life cycle of blocking. During this interaction, the amplification of the wavenumber-2 component is more important for the establishment of a blocking circulation than the details of the other two waves. In addition, a diagnostic study is made so as to confirm our theory. The present paper is organized as follows. In section 2, the derivation of three coupling amplitude equations for quasi-resonant triad planetary waves, forced by synoptic-scale eddies, is described. Numerical solutions to the three equations are presented in section 3. In section

4, we present some theoretical results of the interaction between quasi-resonant triad planetary waves and synoptic-scale waves. The comparison between theoretical results and observational evidence is examined in section 5. We present the main conclusions in section 6.

2. The equivalent barotropic model and governing equations

2.1 Equivalent barotropic model and scale decomposition

Berggren et al. (1949) first noted that the persistence of blocking is associated with the absorption of anticyclonic and cyclonic eddies into the northern and southern halves of a blocking pattern, respectively. However, no acceptable analytical theory was proposed to describe such a process before Haines and Marshall (1987) investigated this problem numerically. Malguzzi (1993) proposed an analytical theory to investigate the relationship between a blocking dipole and synoptic eddies. However, as noted in that paper, the role of synoptic-scale eddies in the dynamics of blocking needs further examination. Mathematical description of the block-eddy interaction process is expected to require the model used to be as simple as possible. Consequently, in the present paper we will choose an equivalent barotropic model (Loesch, 1977) to examine the interaction between planetary-scale waves and synoptic-scale waves during the life cycle of blocking.

In the absence of dissipation, the nondimensional, equivalent barotropic vorticity equation with an external vorticity source can be written as

$$\frac{\partial}{\partial t}(\nabla^2\Psi_T - F\Psi_T) + J(\Psi_T, \nabla^2\Psi_T) + \beta \frac{\partial\Psi_T}{\partial x} = \nabla^2\Psi^*, \quad (1)$$

where ψ_T is the total atmospheric streamfunction, $F = (L/R_d)^2$, and $\beta = \beta_0 L^2/U$, in which R_d is the radius of the Rossby deformation, β_0 is the meridional gradient of the Coriolis parameter, and L and U are the horizontal length and velocity scales, respectively; $J(a, b)$ is the Jacobian operator, ∇^2 is the horizontal Laplacian operator, and Ψ^* is the external vorticity source which is assumed to be of synoptic-scale in the present paper. It is shown that when $\Psi^* = 0$, Eq. (1) reduces to that used by Loesch (1977).

For convenience, a scale decomposition is made similar to Luo (2000, 2005a) and Colucci (2001) but without including the medium waves. If ψ and ψ' represent planetary- and synoptic-scale components, respectively, then under the background of a constant westerly wind \bar{u} and for a decomposition of

$\Psi_T = -\bar{u}y + \psi + \psi'$, Eq. (1) can be rewritten as

$$\begin{aligned} & \left[\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) (\nabla^2\psi - F\psi) + J(\psi, \nabla^2\psi)_P + \right. \\ & \quad (\beta + F\bar{u}) \frac{\partial\psi}{\partial x} + J(\psi', \nabla^2\psi')_P + \\ & \quad \left. J(\psi', \nabla^2\psi)_P + J(\psi, \nabla^2\psi')_P \right] + \\ & \left[\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) (\nabla^2\psi' - F\psi') + \right. \\ & \quad J(\psi', \nabla^2\psi')_S + (\beta + F\bar{u}) \frac{\partial\psi'}{\partial x} + \\ & \quad \left. J(\psi, \nabla^2\psi)_S + J(\psi', \nabla^2\psi)_S + J(\psi, \nabla^2\psi')_S \right] \\ & = \nabla^2\Psi^*, \end{aligned} \quad (2)$$

where $J(\psi, \nabla^2\psi)$, $J(\psi', \nabla^2\psi)$, and $J(\psi, \nabla^2\psi')$ have been decomposed into planetary-scale $J(\psi, \nabla^2\psi)_P$, $J(\psi', \nabla^2\psi)_P$, and $J(\psi, \nabla^2\psi')_P$ and synoptic-scale $J(\psi, \nabla^2\psi)_S$, $J(\psi', \nabla^2\psi)_S$, and $J(\psi, \nabla^2\psi')_S$. In addition, $J(\psi', \nabla^2\psi')$ has been separated into two parts: $J(\psi', \nabla^2\psi')_P$ and $J(\psi', \nabla^2\psi')_S$. Note that the subscript “P” denotes the planetary-scale component, but the subscript “S” represents the synoptic-scale component. The same subscripts are still used hereafter.

In this paper, we consider ultra long planetary-scale waves as a superposition of zonal wavenumbers 1–3 because the blocking flow is mainly comprised of zonal wavenumbers 1–3 (Colucci et al., 1981). In a weak background westerly wind environment, the synoptic-scale waves of less than six days period generally have zonal wavenumbers larger than 9 (Luo, 2000, 2005a). It is easy to find that for zonal wavenumbers 1–3, the zonal wavenumber of $J(\psi, \nabla^2\psi)$ is less than 5. This implies that $J(\psi, \nabla^2\psi)$ is planetary-scale or at least medium-scale. That is, the projection of $J(\psi, \nabla^2\psi)$ onto the synoptic-scale almost disappears. In this case,

$$J(\psi, \nabla^2\psi)_S \approx 0$$

and

$$J(\psi, \nabla^2\psi) \approx J(\psi, \nabla^2\psi)_P.$$

For example, for two Rossby waves of zonal wavenumbers 2 (k_2) and 3 (k_3), $J(\psi, \nabla^2\psi)$ has a wavenumber-1 component ($k_3 - k_2$) and a wavenumber-5 component ($k_2 + k_3$). In other words, the planetary-scale of $J(\psi, \nabla^2\psi)$ is dominant. If the prescribed synoptic-scale waves have prominent zonal wavenumbers larger than 9, then the projection of $J(\psi, \nabla^2\psi')$ and $J(\psi', \nabla^2\psi)$ onto the planetary-scale (1–3) almost

vanishes. In this case,

$$\begin{aligned} J(\psi', \nabla^2 \psi')_P &\approx 0, \\ J(\psi, \nabla^2 \psi')_P &\approx 0, \\ J(\psi', \nabla^2 \psi) &\approx J(\psi', \nabla^2 \psi)_S, \end{aligned}$$

and

$$J(\psi, \nabla^2 \psi') \approx J(\psi, \nabla^2 \psi')_S.$$

Based upon the above considerations, we can obtain from Eq. (2) (Luo, 2000, 2005a)

$$\begin{aligned} &\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) (\nabla^2 \psi - F\psi) + \\ &J(\psi, \nabla^2 \psi) + (\beta + F\bar{u}) \frac{\partial \psi}{\partial x} \\ &= -J(\psi', \nabla^2 \psi')_P, \end{aligned} \quad (3a)$$

$$\begin{aligned} &\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) (\nabla^2 \psi' - F\psi') + \\ &(\beta + F\bar{u}) \frac{\partial \psi'}{\partial x} + J(\psi', \nabla^2 \psi')_S \\ &= -J(\psi', \nabla^2 \psi) - J(\psi, \nabla^2 \psi') + \nabla^2 \Psi^*, \end{aligned} \quad (3b)$$

where $-J(\psi', \nabla^2 \psi')_P$ represents the net forcing of synoptic eddies and acts as a source of energy for the incipient blocking pattern (Holopainen and Fortelius, 1987). The above scale decomposition is crudely acceptable for the study of the physical mechanism of blocking dynamics (Luo, 2005a). This is mainly based upon an important assumption that planetary waves must be long enough and synoptic-scale waves must be short enough: both $J(\psi, \nabla^2 \psi')$ and $J(\psi', \nabla^2 \psi)$ are approximately synoptic-scale waves for very long planetary waves equal or less than wavenumber 3 and for very short synoptic-scale waves with zonal wavenumbers larger than 9. In the mid-high latitudes, for stationary Rossby waves for blocking to be excited requires weak background westerly winds. For example, the background westerly wind for a stationary wavenumber 2 with dipole meridional structure is 7 ms^{-1} according to the dispersion relation of barotropic Rossby waves. In this case, the zonal wavenumbers of synoptic-scale waves with periods less than one week must be 9 or larger. Thus, the above scale decomposition has a firm theoretical basis. However, real blocking cases do not strictly satisfy such scale decomposition. As in Luo (2000), $J(\psi, \nabla^2 \psi')$ and $-J(\psi', \nabla^2 \psi)$ should include zonal wavenumbers 7, 9, 11, and 13 if zonal wavenumber 2 is considered as a planetary wave and the synoptic-scale waves are represented by the superposition of zonal wavenumbers 9 and 11. Thus

these assumptions that $J(\psi, \nabla^2 \psi')$ and $J(\psi', \nabla^2 \psi)$ are synoptic-scale components are approximately acceptable for studying the interaction between planetary-scale diffluent flow and synoptic-scale waves related to blocking onset. Actually, including $J(\psi', \nabla^2 \psi')$ s in Eq. (3b) is more reasonable, but $J(\psi', \nabla^2 \psi')$ s is in fact a small term. Thus, it is concluded that Eqs. (3a–b) can be regarded as the extension of the time-mean block-eddy interaction model. For weak background westerly winds, the zonal wavelength of Rossby waves must be required to be shorter so as to make the Rossby waves be synoptic-scale waves with periods less than one week. In this case, the time and space filters have almost the same meaning. But the spatial decomposition of the equation of motion is better than its time decomposition (Haines and Marshall, 1987).

As indicated by some investigators (Colucci et al., 1981), blocking is dominated by zonal wavenumbers 1–4, and especially by zonal wavenumbers 1–3. Thus, the onset and decay of blocking is associated with the increase and decrease of the amplitudes of the four waves forced directly by synoptic-scale waves with short wavelengths. For most blocking cases, medium-scale waves do not assist in the planetary-scale amplification of blocking, as demonstrated in section 5 of this paper, and the synoptic-scale waves do not force the medium-scale waves (wavenumbers 5–7) during the blocking period even though these medium-scale waves may affect the synoptic-scale waves.

Thus, in the diagnostic study of a blocking event, the medium waves should be excluded so as to correctly evaluate the contributions of the planetary-to-synoptic-scale interaction (PSI) terms. The synoptic-scale projection of the PSI terms must be dominant if the medium waves are filtered out. Inclusion of the medium waves in the theoretical studies will make it difficult for us to disentangle the real contributions of both the PSI terms and the eddy forcing associated with blocking onset. The classic work of Hansen and Chen (1982) confirmed that the interactions between planetary-scale waves (with wavenumbers 1–4) and all other waves contribute importantly to the planetary-scale kinetic energy during an analyzed blocking episode. However, the diagnostic study of Franzke et al. (2000, their Fig. 8) emphasized the role of the high-frequency eddy forcing in blocking onset, rather than the role of the PSI. In fact, the enhanced planetary-scale kinetic energy, due to the interactions between planetary-scale waves (with wavenumbers 1–4) and all other waves, is a result of blocking establishment caused by the eddy forcing. It is easily explained: Following Hansen and Chen (1982), it is inevitable that the interaction term

$$-J(\psi'_{5-10}, \nabla^2 \psi_{1-4}) - J(\psi_{1-4}, \nabla^2 \psi'_{5-10}) = J_{1-4}$$

possesses planetary-scale components with zonal wavenumbers 1–4. Because the blocking waves (1–4) are amplified by the synoptic-scale waves, the planetary-scale kinetic energy due to J_{1-4} will have an enhanced trend during the blocking onset. In this case, the change of the eddy induced planetary-scale kinetic energy due to J_{1-4} is in phase with the kinetic energy of growing planetary-scale blocking waves (1–4). The result presented by Hansen and Chen (1982, Fig. 5a) clearly shows this point. At the same time, we can find from their Fig. 5b that the eddy kinetic energy decreases with the growth of the blocking kinetic energy. This further indicates that the blocking may occur through the upscale transfer of eddy kinetic energy (through high frequency eddy forcing). In fact, the medium-scale waves do not assist in the planetary-scale amplification of blocking. The onset of blocking circulation is mainly attributed to the high-frequency eddy forcing, as demonstrated in some diagnostic studies (Holopainen and Fortelius, 1987; Franzke et al., 2000).

Therefore, in the diagnostic study of blocking, the interactions between planetary-scale waves (with wavenumbers 1–4) and all other waves, computed from real blocking events, will, to large extent, make it more difficult to disentangle the dynamics, and will mask the real roles of the synoptic-scale waves and the planetary-to-synoptic-scale interaction in exciting a blocking flow if the medium waves are included. In this paper, ignoring the medium waves leads to three good features: (1) the role of pure synoptic-scale waves in exciting the blocking flow can be easily explored; (2) the theoretical model can be greatly simplified; (3) the analytical solution of the feedback of blocking diffluent flow on synoptic-scale waves can be easily obtained by neglecting the medium-scale waves in our model. Consequently, it can be concluded that the fact that the PSI term does not appear in Eq. (3a) is not only mathematically realistic, but also physically realistic. Neglecting the medium-scale waves can accentuate more important terms contributing to blocking onset. This formalism does not contradict the diagnostic result of Hansen and Chen (1982). Unfortunately, the diagnostic result of Hansen and Chen (1982) that the planetary (1–4)-to-synoptic (5–10)-scale interaction terms possess a large planetary-scale component is easily misunderstood to play an important role in blocking onset. Actually, only the planetary-scale projection of the self-interaction among the synoptic-scale waves plays a key role for blocking onset (Holopainen and Fortelius, 1987; Franzke et al., 2000).

The planetary-scale component $-J(\psi', \nabla^2 \psi')_P$ in Eq. (3a) can be considered as an external forcing of the blocking wave (ψ), which will be referred to

as “eddy forcing” hereafter (Colucci, 1985; Malguzzi, 1993) or planetary-scale projection of self-interaction among synoptic-scale eddies (Colucci, 2001). In the diagnostic study of Franzke et al. (2000), $-J(\psi', \nabla^2 \psi')_P$ was termed as “high-frequency eddy forcing”. It can be seen from Eq. (3a) that the alteration of the planetary-scale wave ψ is mainly induced by the eddy forcing $-J(\psi', \nabla^2 \psi')_P$, while the alteration of the synoptic-scale eddies (ψ') is mainly caused by planetary-(synoptic-) to synoptic-(planetary-) scale interactions $J(\psi, \nabla^2 \psi')$ and $J(\psi', \nabla^2 \psi)$ that represent the feedback of developing blocking waves (planetary-scale diffluent flow) on synoptic eddies. The above decomposition can show different roles of planetary-scale projections of self-interaction among synoptic-scale eddies and planetary-(synoptic-) to synoptic- (planetary-) scale interactions in exciting blocking flow (Colucci, 2001).

The planetary-scale flow is contained within a beta-channel on whose boundaries (Loesch, 1977)

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial^2 \bar{\psi}}{\partial t \partial y} = 0, \quad y = 0, L_y, \quad (4)$$

where L_y is the width of the beta-channel, and $\bar{\psi}(y, t)$ is the zonally averaged part of the streamfunction.

2.2 Governing equations of three quasi-resonant waves during their interaction

It should be noted that when the eddy forcing is not involved in Eq. (3a), it reduces to the model considered by Loesch (1977). Colucci (1985) had shown that atmospheric blocking patterns arise due to the interaction of transient, synoptic-scale perturbations with the planetary-scale environment. In this context, blocking may be understood as a response of planetary-scale waves to synoptic-scale waves that act as sources of energy and vorticity for the incipient blocking, which has been examined by Haines and Marshall (1987), Malanotte-Rizzoli and Malguzzi (1987) and Malguzzi (1993) theoretically. However, these theoretical studies cannot provide the instantaneous change of planetary-scale and synoptic-scale fields during the life cycle of a blocking flow. This is an intention of the present paper. It is now known that during the alteration of planetary circulation associated with blocking onset, the synoptic-scale waves tend to split and enhance (Holopainen and Fortelius, 1987; Nakamura and Wallace, 1990, 1993). This shows that the blocking circulation and the synoptic-scale eddies have a symbiotic relation (Cai and Mak, 1990). However, to leading order approximation the blocking circulation behaves as “free quasi-stationary wave” (Holopainen and Fortelius, 1987). Thus, it is natural

to conclude that during the coupling between blocking and synoptic-scale waves, the amplitudes of planetary waves at the leading order approximation should at least be “slowly varying” in both space and time. In this case, the contribution of synoptic-scale eddies to the development of blocking waves can be described by the slowly varying equations of planetary waves forced by synoptic-scale eddies at the second order.

Based upon the above consideration, we introduce the following slow time and space variables

$$T = \varepsilon t, \quad X = \varepsilon x, \quad (5)$$

where ε is a small parameter and $0 < \varepsilon \ll 1.0$.

In this paper, we assume that the synoptic-scale vorticity source is weak, so that it can balance the slowly varying terms of the preexisting synoptic-scale eddies. In this case, we suppose $\Psi^* = \varepsilon^2 \Psi_1^*$.

To treat the coupling between planetary- and synoptic-scale waves, we may expand planetary-scale waves (ψ) and synoptic-scale waves (ψ') as $\psi = \varepsilon(\psi_1 + \varepsilon\psi_2 + \dots)$ and

$$\psi' = \varepsilon\psi'_1(x, y, t, X) + \varepsilon^2\psi'_2(x, y, t, T) + \dots,$$

respectively. Thus, it follows from Eq. (3b) that

$$\begin{aligned} N(\psi'_1) &= \left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) [\nabla^2(\psi'_1) - F(\psi'_1)] + \\ &(\beta + F\bar{u}) \frac{\partial(\psi'_1)}{\partial x} = 0, \end{aligned} \quad (6a)$$

$$\begin{aligned} N(\psi'_2) &= -J(\psi'_1, \nabla^2\psi_1) - J(\psi_1, \nabla^2\psi'_1) - \\ &J(\psi'_1, \nabla^2\psi'_1)_S - \bar{u} \frac{\partial}{\partial X} \nabla^2\psi'_1 - \\ &2\bar{u} \frac{\partial^3\psi'_1}{\partial x^2 \partial X} - \beta \frac{\partial\psi'_1}{\partial X} + \nabla^2\Psi_1^*, \end{aligned} \quad (6b)$$

where $N()$ is the linear operator of the Rossby wave.

Note that ψ'_1 represents the first order approximation to the synoptic-scale waves and is referred to as representing preexisting synoptic-scale eddies fixed upstream of an incipient blocking, and ψ'_2 is the second order correction to the preexisting synoptic-scale waves which represents the feedback of developing blocking on preexisting synoptic-scale eddies. It should be pointed out that if ψ'_1 and ψ'_2 possess the same synoptic-scale, then the slowly varying terms of synoptic-scale eddies and $\bar{\psi}_1^*$ in Eq. (3) will be secular terms. To cancel the secular terms, we can allow

$$\bar{u} \frac{\partial}{\partial X} \nabla^2\psi'_1 + 2\bar{u} \frac{\partial^3\psi'_1}{\partial x^2 \partial X} + \beta \frac{\partial\psi'_1}{\partial X} = \nabla^2\Psi_1^*. \quad (7)$$

It is clear that ψ'_1 can be derived if Ψ_1^* is prescribed. Of course, if ψ'_1 is prescribed, Ψ_1^* can be obtained.

Thus, it is sufficient to give the mathematical expression of ψ'_1 in studying the interaction of an incipient blocking with synoptic-scale eddies.

The above expansions mean that the eddy forcing, $-J(\psi', \nabla^2\psi')_P$, induced by synoptic eddies have the same order as the planetary-scale advection term $-J(\psi, \nabla^2\psi)$ in Eq. (3a). This is mainly based upon observational evidence that the “climatological average” winter eddy vorticity forcing is generally two or three times smaller than the mean (planetary-scale) vorticity flux divergence observed by Holopainen (1978) and Lau (1979). Of course, it is not strict for all blocking events.

Rex (1950a, 1950b) observed that the formation of a blocking anticyclone is initiated by a “finite external-impulse” provided by a sufficiently intense cyclonic disturbance. Hansen and Chen (1982) found the transport of energy and enstrophy from cyclone-scale eddies to planetary-scale waves (zonal harmonic wavenumbers 1–3) through a nonlinear upscale energy cascade during the blocking periods. In this process, the amplitude of planetary-scale waves could be more or less regarded as being slowly varying in time and space due to the eddy forcing produced by upstream synoptic-scale waves.

The solution to Eq. (3a) is expanded as

$$\psi = \sum_{n=1}^{\infty} \varepsilon^n \psi_n(x, y, t, T, X). \quad (8)$$

Substitution of Eq. (5) and (8) into Eq. (3a) yields

$$\begin{aligned} O(\varepsilon^1) : N(\psi_1) &= \left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) [\nabla^2(\psi_1) - F(\psi_1)] + \\ &(\beta + F\bar{u}) \frac{\partial(\psi_1)}{\partial x} = 0, \end{aligned} \quad (9a)$$

$$\begin{aligned} O(\varepsilon^2) : N(\psi_2) &= - \left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) [\nabla^2(\psi_1) - F(\psi_1)] - \\ &2 \left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) \frac{\partial^2\psi_1}{\partial x \partial X} - (\beta + F\bar{u}) \frac{\partial\psi_1}{\partial X} - \\ &J(\psi_1, \nabla^2\psi_1) - J(\psi'_1, \nabla^2\psi'_1)_P. \end{aligned} \quad (9b)$$

It is clear from Eq. (9a) that to leading order approximation the blocking behaves as linear Rossby waves, but its alteration can be described by the slowly modulated amplitude equations of linear Rossby waves that are only derived at the second-order approximation.

As an example, three Rossby waves for zonal wavenumbers 1 (monopole), 2 (dipole), and 3 (monopole) are considered in this paper. In this case,

the solution to (9a) can be assumed to be a superposition of the following three Rossby waves

$$\begin{aligned}\psi_1 &= A_1(T, X) \exp[i(k_1 x - \omega_1 t)] \sin(my) + \\ &A_2(T, X) \exp[i(k_2 x - \omega_2 t)] \sin(2my) + \\ &A_3(T, X) \exp[i(k_3 x - \omega_3 t)] \sin(my) + \text{cc} ,\end{aligned}\quad (10)$$

where

$$\begin{aligned}\omega_i &= \bar{u}k_i - \frac{(\beta + F\bar{u})k_i}{k_i^2 + m^2 + F} \quad (i = 1, 3) , \\ \omega_2 &= \bar{u}k_2 - \frac{(\beta + F\bar{u})k_2}{k_2^2 + 4m^2 + F} , \\ m &= -\pi/L_y , k_1 = k_0 , k_2 = 2k_0 , k_3 = 3k_0 , \\ k_0 &= 1/[6.371 \cos(\phi_0)]\end{aligned}$$

is the zonal of wavenumber one, ϕ_0 is the reference latitude, and $A_n(T)$ ($n = 1, 2, 3$) represents the slowly varying complex amplitude of the n th linear Rossby wave and cc denotes the complex conjugate of its preceding term.

In this paper, we will consider the superposition of quasi-resonant triad waves as a model of an incipient blocking. This consideration is mainly based upon a major point: The incipient blocking prior to blocking onset is commonly observed to be a blocking ridge. Thus, it might be a superposition of triad resonant planetary waves having monpole and dipole meridional structures (Colucci et al., 1981). Excluding the triad interaction will reduce the realism of blocking pattern. Colucci et al. (1981) emphasized that triad interaction is an important mechanism for blocking onset. However, in this paper the eddy forcing is stressed to play a crucial role for blocking onset.

In the first few days of blocking onset, intense, developing upstream synoptic-scale waves precede the growth of blocking. Thus, following Luo (2000) the preexisting synoptic-scale waves upstream can be approximately assumed to be described by

$$\begin{aligned}\psi'_1 &= f'_0(X) \{ \exp[i(\tilde{k}_1 x - \tilde{\omega}_1 t)] - \\ &\exp[i(\tilde{k}_2 x - \tilde{\omega}_2 t)] \} \sin(my) + \text{cc} ,\end{aligned}\quad (11)$$

where

$$\begin{aligned}\tilde{\omega}_1 &= \bar{u}\tilde{k}_1 - \frac{(\beta + F\bar{u})\tilde{k}_1}{\tilde{k}_1^2 + m^2 + F} , \\ \tilde{\omega}_2 &= \bar{u}\tilde{k}_2 - \frac{(\beta + F\bar{u})\tilde{k}_2}{\tilde{k}_2^2 + m^2 + F} , \\ \tilde{k}_1 &= (n - \Delta n)k_0 , \quad \tilde{k}_2 = (n + \Delta n)k_0 ,\end{aligned}$$

and f'_0 is a localized function which represents the amplitude distribution of upstream synoptic-scale waves.

It should be noted that for weak background westerly winds, synoptic-scale waves require $n \geq 10$ (Luo,

2000). For example, with $\bar{u} = 0.7$, $n = 10$, and $\Delta n = 0.75k_0$, the Rossby waves in Eq. (11) are of synoptic-scale and have periods of less than one week. Of course, Eq. (11) can represent synoptic-scale waves if other parameter values are chosen.

In the mid-high latitudes, weak background westerly winds prevail over the North Pacific and Atlantic oceans. As pointed out by Shutts (1983), this condition is a precondition for blocking onset. If $L_y = 5.0$, $F = 1.0$, and $\bar{u} = 0.7$ are chosen,

$$\omega_3 - \omega_2 - \omega_1 = \Delta\omega = -0.0415$$

is satisfied at 55°N . Of course, this quasi-resonant condition is also tenable for other parameters in a moderate range. In this case, a quasi-resonant interaction occurs for the three Rossby waves as described in Eq. (10) (Craik, 1985). The theoretical and numerical studies of Luo (2000) and Franzke et al. (2000) demonstrated that “spatial resonance” of zonal wave 2 and high-frequency eddy forcing is the amplifying mechanism for block onset. Based upon this idea, we consider that the zonal wavenumber of the planetary-scale component

$$\tilde{k}_2 - \tilde{k}_1 [J(\psi', \nabla^2 \psi')_P]$$

is near or equal to the zonal wavenumber (k_2) of zonal wave 2. For example, for $\bar{u} = 0.7$, $n = 10$, and $\Delta n = 0.75$ it is easy to obtain

$$\tilde{k}_2 - \tilde{k}_1 - k_2 = -0.137$$

and

$$\tilde{\omega}_2 - \tilde{\omega}_1 - \omega_2 = \Delta\Omega = 0.351$$

at 55°N . In this case, the eddy forcing can near-resonantly force zonal wave 2. Thus, if $\varepsilon = 0.24$ is chosen, it is reasonable to write $\Delta\omega = \varepsilon\Delta\omega_0$ and $\Delta\Omega = \varepsilon\Delta\Omega_0$, approximately. In particular, we also note that if $\Delta n = 1$ is chosen, there is the condition $\tilde{k}_2 - \tilde{k}_1 - k_2 = 0$. In this case, the so-called “spatial resonance” is strictly satisfied, but the basic conclusion is similar.

Substitution of Eqs. (10) and (11) into (9b) yields the following three equations for the solvability condition

$$\frac{\partial A_1}{\partial T} + C_{g1} \frac{\partial A}{\partial X} = iS_1 A_3 A_2^* \exp(-i\Delta\omega_0 T) , \quad (12a)$$

$$\begin{aligned}\frac{\partial A_2}{\partial T} + C_{g2} \frac{\partial A_2}{\partial X} &= iS_2 A_3 A_1^* \exp(-i\Delta\omega_0 T) + \\ &iGf_0'^2 \exp[i(\kappa X - \Delta\Omega_0 T)] ,\end{aligned}\quad (12b)$$

$$\frac{\partial A_3}{\partial T} + C_{g3} \frac{\partial A_3}{\partial X} = iS_3 A_1 A_2 \exp(i\Delta\omega_0 T) , \quad (12c)$$

where

$$\begin{aligned}
 \tilde{k}_2 - \tilde{k}_1 - k_2 &= \Delta k = \varepsilon \kappa, \\
 S_1 &= \frac{m(2k_3 - k_2)[k_2^2 + 4m^2 - (k_3^2 + m^2)]}{2(k_1^2 + m^2 + F)}, \\
 S_2 &= \frac{m(k_1 + k_3)[k_3^2 + m^2 - (k_1^2 + m^2)]}{2(k_2^2 + 4m^2 + F)}, \\
 S_3 &= \frac{m(2k_1 + k_2)[k_2^2 + 4m^2 - (k_1^2 + m^2)]}{2(k_3^2 + m^2 + F)}, \\
 C_{gi} &= \bar{u} - \frac{(\beta + F\bar{u})(m^2/4 + F - k_i^2)}{(k_i^2 + m^2/4 + F)^2} (i = 1, 3), \\
 C_{g2} &= \bar{u} - \frac{(\beta + F\bar{u})(m^2/4 + F - k_2^2)}{(k_2^2 + m^2/4 + F)^2}, \\
 G &= \frac{(\tilde{k}_1 + \tilde{k}_2)m(\tilde{k}_1^2 - \tilde{k}_2^2)}{k_2^2 + 4m^2 + F},
 \end{aligned}$$

and A_n^* is the complex conjugate of A_n .

Equations (12a–c), which govern the time evolution of triad waves, are used to describe the alteration of an incipient blocking caused by upstream synoptic-scale waves. By adjusting the initial conditions, various numerical solutions to Eqs. (12a–c) can be derived by numerical schemes if the eddy forcing is prescribed. For $f'_0 = 0$, Eqs. (12a–c) are identical to the quasi-resonant three-wave coupling equations that possess soliton solutions in some parameter ranges (Craik, 1985). However, as the eddy forcing is involved, the derivation of the analytical solutions to Eqs. (12a–c) becomes rather difficult. Based upon this consideration, a finite-difference scheme will be used to solve Eqs. (12a–c).

Substituting Eqs. (10) and (11) into Eq. (6b) gives

$$\begin{aligned}
 \psi'_2 &= -\frac{mf'_0 A_1(k_1 - \tilde{k}_1)}{2} \Re_1 \exp\{i[(k_1 + \tilde{k}_1)x - \\
 &\quad (\tilde{\omega}_1 + \omega_1)t]\} \sin(2my) + \\
 &\quad \frac{mf'_0 A_1(k_1 - \tilde{k}_2)}{2} \Re_2 \exp\{i[(k_1 + \tilde{k}_2)x - \\
 &\quad (\tilde{\omega}_2 + \omega_1)t]\} \sin(2my) + \\
 &\quad \frac{mf'_0 A_1^*(k_1 + \tilde{k}_1)}{2} \rho_1 \exp\{i[(\tilde{k}_1 - k_1)x - \\
 &\quad (\tilde{\omega}_1 - \omega_1)t]\} \sin(2my)
 \end{aligned}$$

$$\begin{aligned}
 &\quad - \frac{mf'_0 A_1^*(k_1 + \tilde{k}_2)}{2} \rho_2 \exp\{i[(\tilde{k}_2 - k_1)x - \\
 &\quad (\tilde{\omega}_2 - \omega_1)t]\} \sin(2my) + \frac{mf'_0 A_2}{2} \times \\
 &\quad \exp\{i[(\tilde{k}_1 + k_2)x - (\tilde{\omega}_1 + \omega_2)t]\} [(2\tilde{k}_1 - k_2)J_{11} \sin(3my) - \\
 &\quad (2\tilde{k}_1 + k_2)J_{12} \sin(my)] - \\
 &\quad \frac{mf'_0 A_2}{2} \exp\{i[(\tilde{k}_2 + k_2)x - (\tilde{\omega}_2 + \omega_2)t]\} \times \\
 &\quad [(2\tilde{k}_2 - k_2)J_{21} \sin(3my) - J_{22}(2\tilde{k}_2 + k_2) \sin(my)] + \\
 &\quad \frac{mf'_0 A_2^*}{2} \exp\{i[(\tilde{k}_1 - k_2)x - (\tilde{\omega}_1 - \omega_2)t]\} \times \\
 &\quad [(2\tilde{k}_1 + k_2)S_{11} \sin(3my) - (2\tilde{k}_1 - k_2)S_{12} \sin(my)] - \\
 &\quad \frac{mf'_0 A_2^*}{2} \exp\{i[(\tilde{k}_2 - k_2)x - (\tilde{\omega}_2 - \omega_2)t]\} \times \\
 &\quad [(2\tilde{k}_2 + k_2)S_{21} \sin(3my) - (2\tilde{k}_2 - k_2)S_{22} \sin(my)] - \\
 &\quad \frac{mf'_0 A_3(k_3 - \tilde{k}_1)}{2} \chi_1 \exp\{i[(k_3 + \tilde{k}_1)x - \\
 &\quad (\tilde{\omega}_1 + \omega_3)t]\} \sin(2my) + \frac{mf'_0 A_3(k_3 - \tilde{k}_2)}{2} \times \\
 &\quad \chi_2 \exp\{i[(k_3 + \tilde{k}_2)x - (\tilde{\omega}_2 + \omega_3)t]\} \sin(2my) + \\
 &\quad \frac{mf'_0 A_3^*(k_3 + \tilde{k}_1)}{2} \gamma_1 \exp\{i[(\tilde{k}_1 - k_3)x - \\
 &\quad (\tilde{\omega}_1 - \omega_3)t]\} \sin(2my) + \frac{mf'_0^2 \rho}{2} \sigma \times \\
 &\quad \exp\{i[(\tilde{k}_1 + \tilde{k}_2)x - (\tilde{\omega}_1 + \tilde{\omega}_2)t]\} \sin(2my), \tag{13}
 \end{aligned}$$

where the coefficients of Eq. (13) are given in the Appendix.

It is found from Eq. (13) that the coupling between both preexisting synoptic-scale eddies (ψ'_1) and developing blocking circulation (ψ_1) downstream induces a second order modification (ψ'_2) to preexisting synoptic-scale eddies, which represents the feedback of developing blocking on preexisting synoptic-scale eddies (ψ'_1). In other words, when an incipient blocking is amplified by the preexisting synoptic-scale eddies (ψ'_1), it can also induce an alteration (ψ'_2) of preexisting synoptic eddies through the coupling with ψ'_1 simultaneously. Based on this idea, the interaction between an incipient blocking downstream and synoptic-scale eddies (ψ') can be represented crudely. In addition, we find that the induced eddies (ψ'_2) are strongly associ-

ated with the amplitudes of developing blocking waves and the intensities and zonal wavenumbers of preexisting synoptic-scale eddies (ψ'_1).

3. Numerical solutions for quasi-resonant three-wave coupling equations with eddy forcing

In this section, we will present numerical solutions to Eqs. (12a–c) for prescribed initial conditions. Some investigations have provided evidence that there exist intense, active synoptic-scale eddies upstream of an incipient blocking in the first few days of blocking establishment (Colucci, 1985, 1987). For the localized eddies, their amplitude can be approximated as (Luo, 2000)

$$f'_0 = a'_0 \exp[-\mu(X + \varepsilon x_0)^2], \quad (14)$$

where a'_0 is the amplitude of f'_0 , x_0 represents its location, and $\mu > 0$. When $x_0 > 0$, it represents that the preexisting synoptic-scale eddies are located upstream of the incipient blocking. When $x_0 = 2.87$ is chosen, these eddies are located $\pi/2$ upstream of the incipient blocking.

As shown by Shutts (1983), Kaas and Branstator (1993), and Colucci and Alberta (1996), the weak background westerly wind is a prerequisite condition for blocking establishment. For this case, without the loss of generality, we may take $\bar{u} = 0.7$, $n = 10$, $\Delta n = 0.75$, $\mu = 2.0$, $x_0 = 2.87/2$, $\varepsilon = 0.24$, and $a'_0 = 0.15/\varepsilon$ as the parameters of the interaction between planetary-scale incipient block and synoptic-scale waves at 55°N . For initial amplitudes $A_1(0) = -0.1/\varepsilon$, $A_2(0) = 0.32/\varepsilon$, and $A_3(0) = -0.1/\varepsilon$, the numerical solutions to Eqs. (12a–c) for $|A_1(X, T)|$, $|A_2(X, T)|$, and $|A_3(X, T)|$ in the (X, T) -plane are shown in Fig. 1.

In the absence of forcing, the three Rossby waves for zonal wavenumbers 1, 2, and 3, as described by Eqs. (12a–c), may possess soliton solutions according to Craik (1985). It is further shown that wavenumber 2 grows in amplitude through the quasi-resonant triad interaction with the other two waves, while the amplitudes of wavenumbers 1 and 3 are decreased (not shown). However, the increase of wavenumber 2 in amplitude is too small to create a realistic blocking flow through the superposition of the triad Rossby waves. Colucci et al. (1981) stressed the contribution of triad resonant interactions to blocking onset, but this resonant interaction seems insufficient unless each planetary wave in the triad has large amplitude (not shown).

Figure 1b shows a periodic amplification of wavenumber 2 due to the eddy forcing that is noticeable in Fig. 2, which is capable of generating a blocking flow. At the same time, this wave almost does

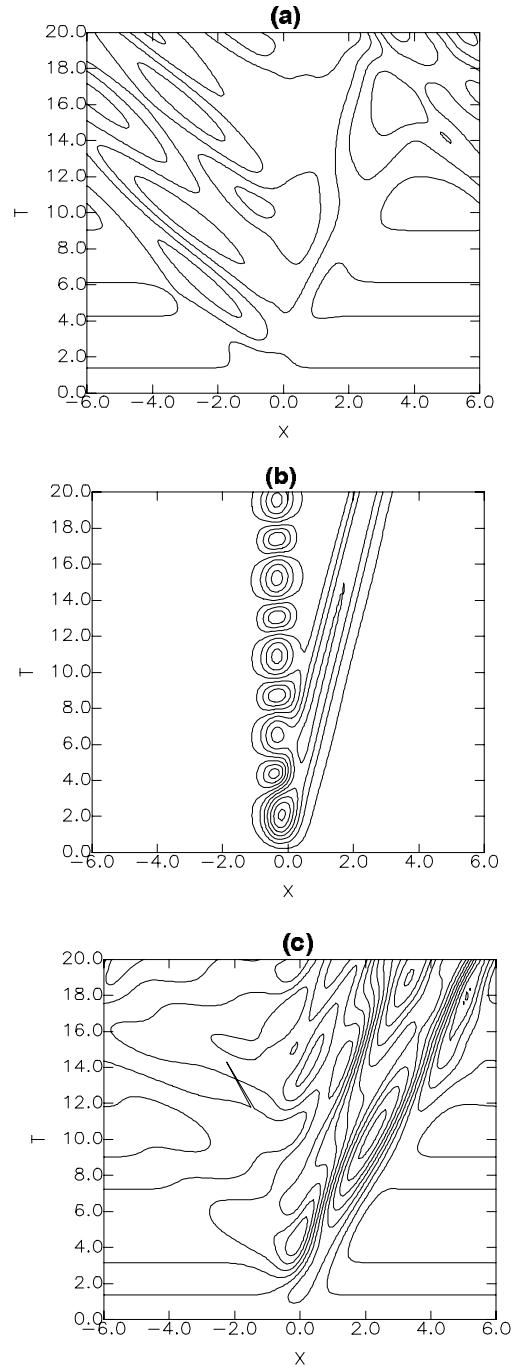


Fig. 1. Time evolution of $|A_1(X, T)|$, $|A_2(X, T)|$, and $|A_3(X, T)|$ in the (X, T) -plane for three Rossby waves for zonal wavenumbers 1 (monopole), 2 (dipole), and 3 (monopole) under the forcing of synoptic-scale waves: (a) $|A_1(X, T)|$. Contour interval: 0.05; (b) $|A_2(X, T)|$. Contour interval (CI) is 0.2; (c) $|A_3(X, T)|$. CI = 0.05.

not propagate, and exhibits a “soliton-oscillation” during its long time evolution. In addition, small amplitude waves propagating eastward can also be excited around it. In this process, the eddy forcing (compo-

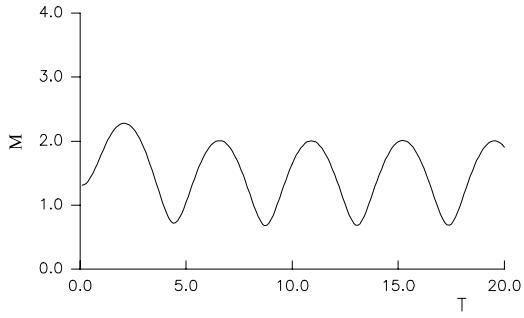


Fig. 2. Time evolution of $|A_2(X, T)|$ at $X = -0.5$, but for the other parameters as in Fig. 1.

ment $\tilde{k}_2 - \tilde{k}_1$) is directly imposed on wavenumber 2 with a dipole meridional structure and causes a periodic amplification of the wavenumber-2 soliton. However, as depicted in Figs. 1a and 1c, because both wavenumbers 1 and 3 have a monpole meridional structure, and they are not directly forced by the eddy forcing, but influenced indirectly by the coupling with wavenumber 2. For this reason, the eddy forcing is not likely to strongly affect wavenumbers 1 and 3. In other words, the role of the amplified wavenumber 2 in causing the establishment of blocking seems to be dominant in comparison with the other two waves. Within a moderate parameter range, this conclusion is not strongly sensitive to the parameter choice (not shown). Of course, the background westerly wind must be weak so that a blocking circulation can be excited (Shutts, 1983).

Figure 2 shows time evolution of

$$M(t) = |A_2(0.5, T)|$$

at $X = 0.5$. It is clear that the oscillation period of the soliton amplitude for $M(t)$ is near $T = 4.5$, which is identical to 21 days in dimensional form. In fact, this describes the period of the transition between high and low indices (blocking) of planetary-scale flow. Since the amplitude of wavenumber 2 is much larger than the others, it actually represents the life cycle of blocking circulation. To show this point, the streamfunction fields of planetary-scale blocking and synoptic-scale waves and their total streamfunction fields will be presented in next section.

4. Life cycle of blocking associated with synoptic-scale waves in our theoretical model and the alteration of subsequent synoptic-scale waves

In this section, we will present the streamfunction fields of triad waves forced by synoptic-scale eddies in order to understand the interaction between blocking

circulation and synoptic-scale eddies. Here, the same parameters as in Fig. 1 are chosen as an example. During the interaction between an incipient blocking and synoptic-scale eddies the instantaneous planetary-scale field ($\psi_p = -\bar{u}y + \psi$ denotes the planetary-scale streamfunction) reconstructed from the sum of triad waves, synoptic-scale field (ψ'), and the total streamfunction field ($\psi_T = \psi_p + \psi'$) are shown in Figs. 3a–c.

It is easy to see from Figs. 3a–b that at the initial stage, the incipient blocking consisting of preexisting triad planetary-scale waves exhibits a weak blocking ridge (incipient blocking) at $x = 0$ and weak synoptic-scale waves exist upstream. The amplification of the incipient blocking caused by upstream synoptic eddies is inevitable because the eddy induced planetary-scale vorticity forcing $[-J(\psi', \nabla^2 \psi'_1)_P]$, as shown in Fig. 4, can provide continuously negative (positive) planetary-scale vorticity transport toward the blocking anticyclonic (cyclonic) region downstream during the onset stage of blocking. This eddy vorticity forcing is a key factor for blocking onset.

As pointed by Colucci (2001), the planetary-scale projection of the self-interaction among synoptic-scale waves contributes more importantly than self-interactions among planetary-scale waves to blocking onset. On the other hand, the synoptic-scale eddies can also alter due to the feedback of amplifying blocking. As described in Fig. 3a, a weak asymmetric dipole is formed at day 3 and continues to strengthen to form a strong asymmetric dipole blocking until day 9. During this period the synoptic-scale waves are enhanced and split into two branches around the blocking region. This behavior is consistent with the observed change of synoptic-scale waves during the life cycle of blocking noted by Holopainen and Fortelius (1987) and by Nakamura and Wallace (1990, 1993). After day 9, as shown in Fig. 4, eddy-induced planetary-scale vorticity transport, opposite to that during the establishment of blocking, begins to take place and leads to the gradual decay of amplified dipole blocking. The dipole blocking may disappear completely by day 21.

Another interesting characteristic is that the synoptic-scale waves become gradually weaker when dipole blocking gradually weakens. Thus, it can be concluded that both the planetary-scale blocking and synoptic-scale waves have a symbiotic relationship during the life cycle of blocking (Cai and Mak, 1990). Figure 3c shows several isolated anticyclonic and cyclonic vortices coexisting within the blocking region, which are in good agreement with those observed by Berggren et al. (1949). The synoptic-scale isolated vortices represent the transient role of synoptic-scale waves in exciting blocking. In the light of this analysis, it can be conjectured that the synoptic-scale waves

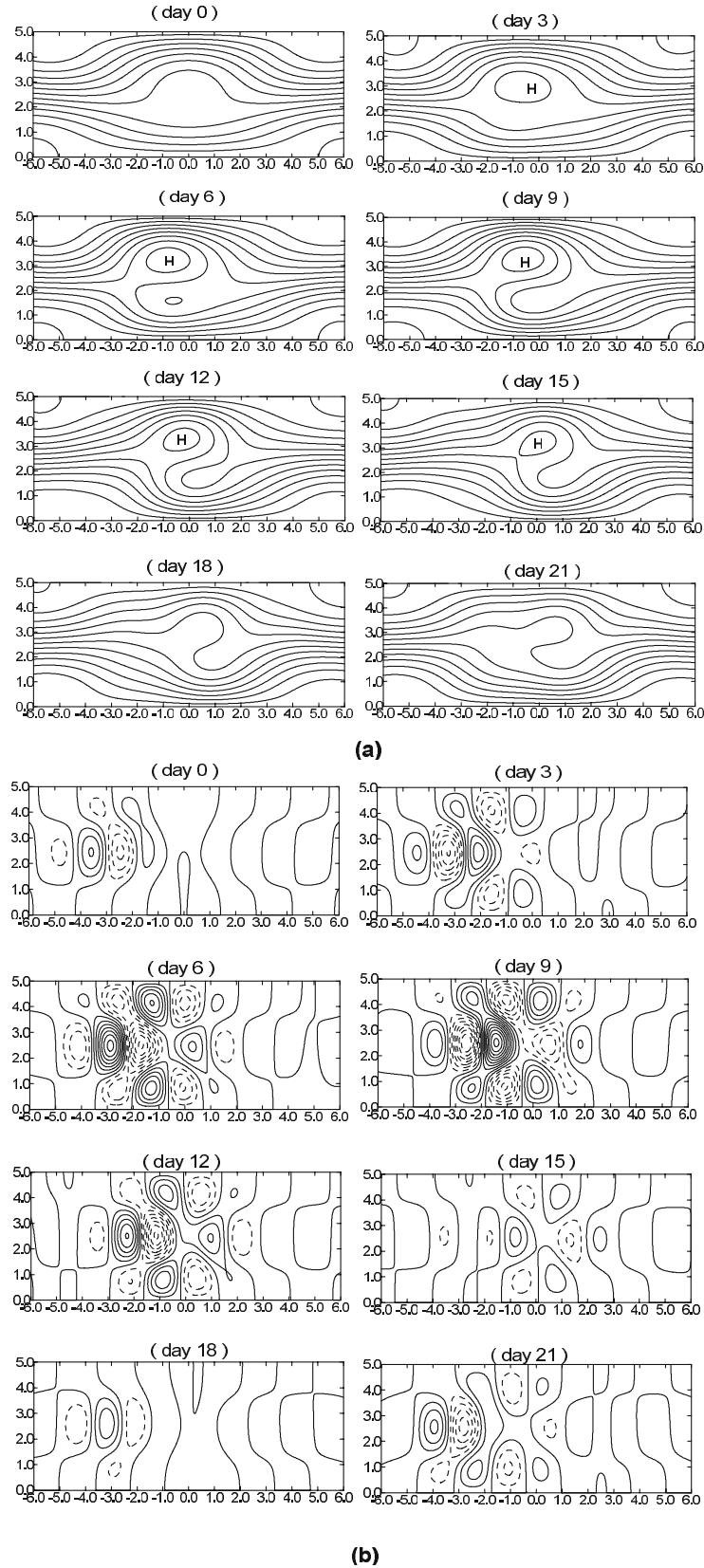


Fig. 3. Instantaneous streamfunction fields of blocking circulation for three quasi-resonant Rossby waves interacting with synoptic-scale waves. The parameters are the same as in Fig. 1. (a) Planetary-scale field (ψ_p): CI=0.3; (b) Synoptic-scale field (ψ'): CI=0.3; (c) Total field (ψ_T): CI=0.3.

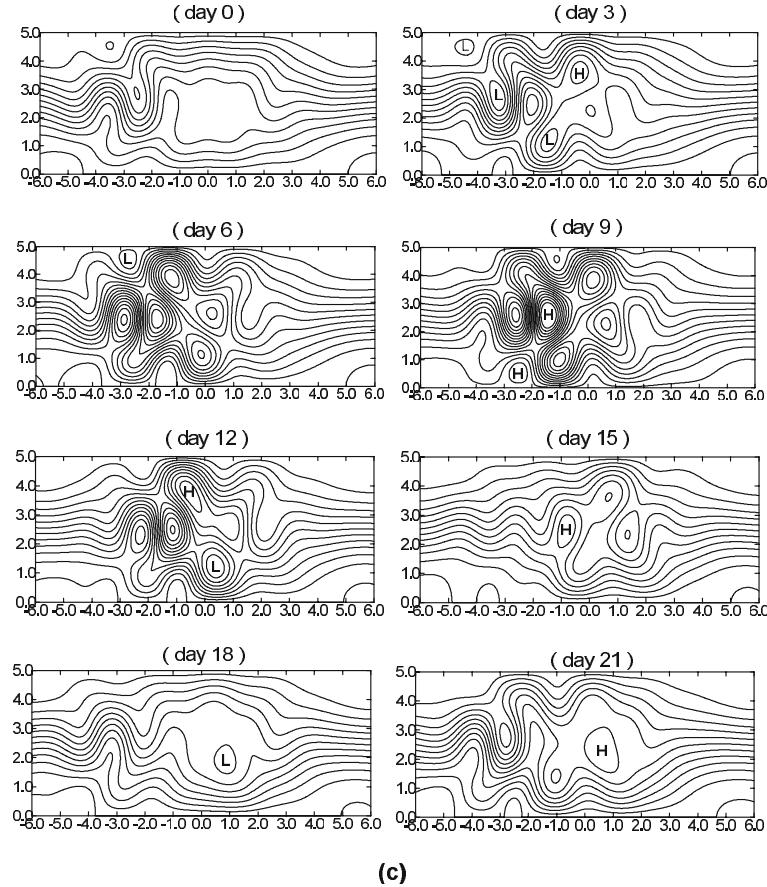


Fig. 3. (Continued).

are acknowledged to play a key role in blocking onset if there are several isolated anticyclonic and cyclonic vortices within the blocking region for observed blocking events. In addition, the time-scale of the theoretical dipole blocking obtained in this paper is found to be about 20 days, which is also consistent with the lifetime of observed blocking. Therefore, our theoretical model can basically capture the interaction between planetary-and synoptic-scale waves during the life cycle of blocking. The interaction between various incipient blocks and synoptic-scale waves can also be understood if and when various initial amplitudes are chosen in our model (not shown).

Here, we will further find that if synoptic-scale waves become more localized, the flow pattern of established blocking will become rather different. For example, Fig. 5 presents the instantaneous field for $\mu = 8.0$.

It is clear in Fig. 5a that an isolated dipole blocking developing from the same incipient blocking, depicted in Fig. 3a, by the upstream, more localized synoptic-scale waves becomes more noticeable, which is slightly different from the mature blocking in Fig. 3a. In addition, the basic characteristics of the changes of the

established dipole blocking and modulated synoptic-scale waves are similar to those in Figs. 3a–b, but the dipole blocking in Fig. 5a seems to become more localized than that in Fig. 3. On the other hand, it appears that the total field in Fig. 3c appears to be slightly different from that shown in Fig. 5c. Thus, this shows that the established blocking pattern by synoptic-scale waves does also depend on the distribution (intensity and location) of preexisting synoptic-scale waves. Furthermore, if one adjusts the background westerly wind and the intensity and location of the preexisting synoptic-scale waves, then the blocking pattern formed may become slightly different (not shown). Although the detailed pattern of established blocking by upstream synoptic-scale waves is sensitive to the distribution (intensity and location) of upstream synoptic-scale waves, the basic conclusion is insensitive to the parameter choices of upstream synoptic-scale waves. Of course, if the synoptic-scale waves are downstream of an incipient blocking, no blocking is excited (figures omitted). That is, the synoptic-scale waves that excite blocking circulation must be located upstream of the preexisting blocking ridge. This confirms the idea of Colucci (1985), who

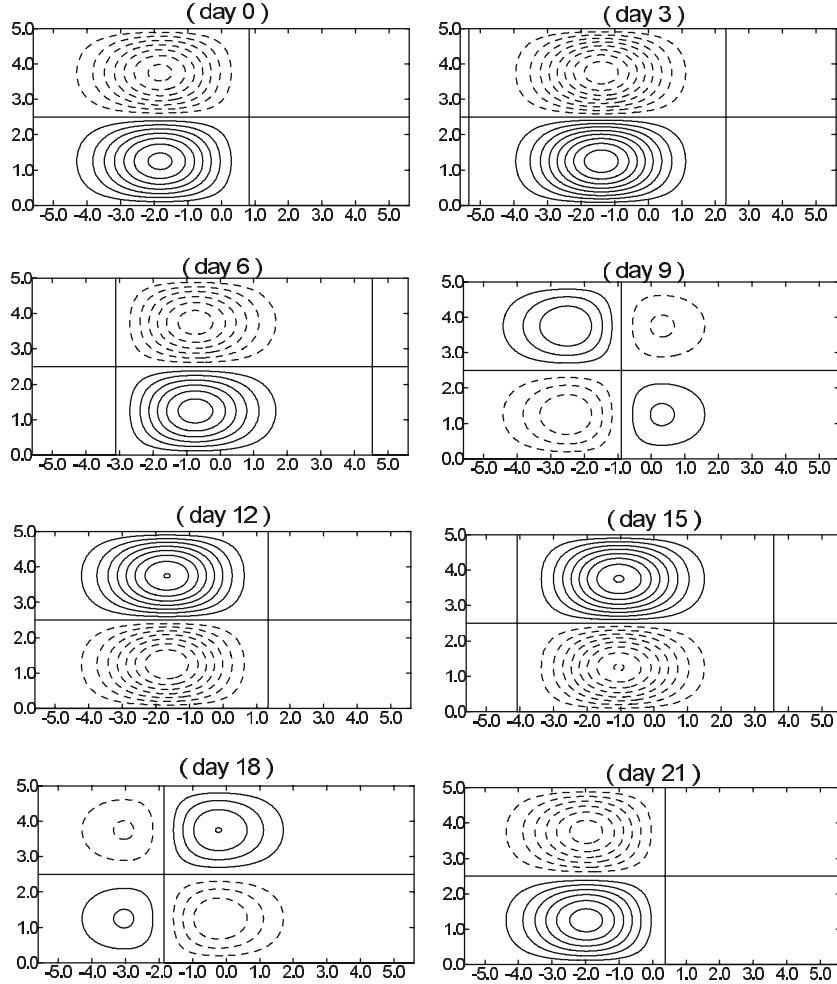


Fig. 4. Instantaneous field of $-J(\psi', \nabla^2\psi'_1)_P$ for parameters $\bar{u} = 0.7$, $n = 10$, $\Delta n = 0.75$, $\mu = 2.0$, $x_0 = 2.87/2$, $\varepsilon = 0.24$, and $a'_0 = 0.15/\varepsilon$. The solid curve denotes positive values and the dashed curve represents the negative values. CI = 0.02.

suggested that both the intensity and location of synoptic-scale perturbations relative to the planetary waves might determine what type of response occurs in the blocking process. Consequently, the quasi-resonant triad interaction theory proposed here can basically represent the life cycle of blocking related to synoptic-scale eddies in observed blocking events. A case study of a blocking event presented in the next section will at least indicate that the above mechanism is possible for some of blocking events.

In our model, the eddy forcing $[-J(\psi'_1, \nabla^2\psi'_1)_P]$ induced by the first order (preexisting) synoptic eddies (ψ'_1) influences (reinforces) planetary-scale blocking flow (ψ_1), and the feedback of planetary-scale flow (ψ_1) on the synoptic-scale eddies (ψ'_1) is mainly represented by ψ'_2 which is effect of the induced eddies by the coupling between the planetary-scale flow (ψ_1) and the first order eddies (ψ'_1). In this process, the

first order eddies (ψ'_1) is not influenced by planetary-scale flow (ψ_1). Thus, they may be seen as free eddies. The alteration of synoptic-scale eddies, displayed in Fig. 3b, is solely due to the contribution of the induced eddies (ψ'_2). In addition, we notice that induced eddies (ψ'_2) get so large during the blocking episode that the expansion, $\psi' = \varepsilon\psi'_1 + \varepsilon^2\psi'_2 + \dots$, may be violated in our model. This is mainly because the wavenumber-2 component that dominates blocking flow has large amplitude, and preexisting synoptic-scale eddies are of shorter zonal scale. According to Eq. (13), ψ'_2 tends to be largest during the mature stage of blocking because its amplitude is proportional to the amplitude (A_2 or A_2^*) of blocking. In this case, ψ'_2 may have the same order as, or even exceed, ψ'_1 . If we consider a weak blocking and weak synoptic-scale eddies having longer zonal wavelength, then ψ'_2 may be smaller in magnitude than ψ'_1 (figures omitted). For

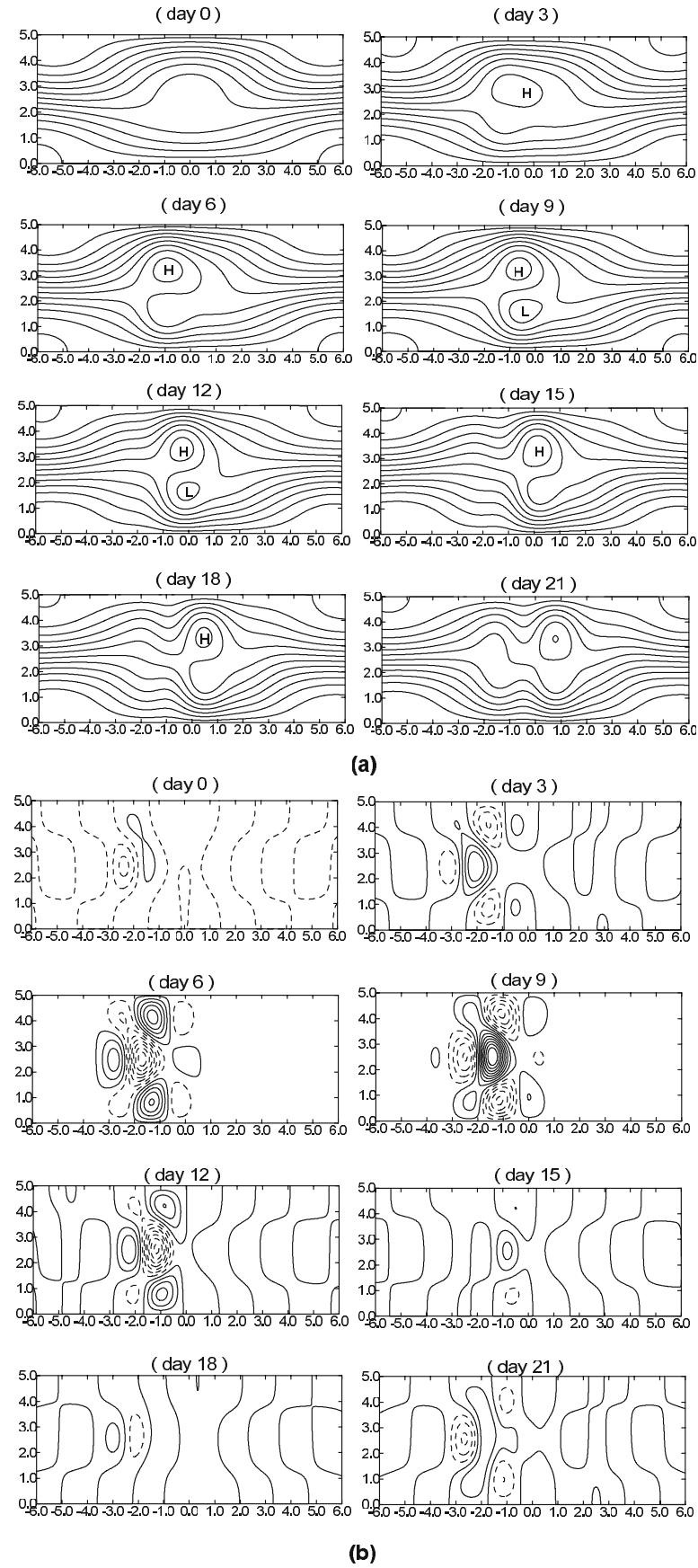


Fig. 5. Same as in Fig. 3, but for $\mu = 8$.

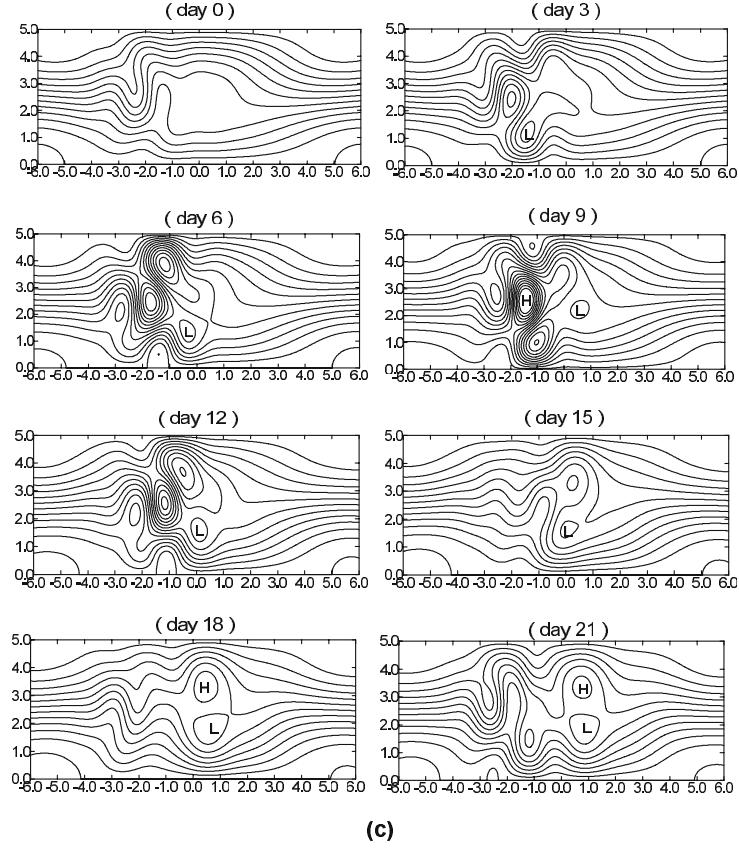


Fig. 5. (Continued).

this case, the changes of both planetary-scale blocking flow and synoptic-scale eddies are also similar to Figs. 3a–c (figures omitted). Despite that the separation of synoptic-scale eddies into free eddies (ψ'_1) and induced eddies (ψ'_2) is not strict from the mathematical point of view, this separation can always capture the essence of the interaction between blocking flow and synoptic-scale eddies.

5. A comparison between the transient forced quasi-resonant wave-interaction theory and a blocking case

In this section, we will justify the transient forced quasi-resonant wave-interaction theory proposed here by using a diagnostic study of a blocking event. The data used in the present study consists of 500-hPa geopotential height data of the NCEP/NCAR reanalysis on a latitude-longitude grid ($2.5^\circ \times 2.5^\circ$). The full grid extends from 30° to 80°N and westward from 100°W to 80°E .

In the present paper, the geostrophic streamfunction was calculated at each grid point (i, j) as $\psi_{i,j} = gZ_{i,j}/\tilde{f}_0$, in which \tilde{f}_0 is the Coriolis parameter centered at 55°N , g is the gravitational acceleration, and

$Z_{i,j}$ is the geopotential height of the isobaric surface. The instantaneous fields of geostrophic streamfunctions (1200 GMT) during the life cycle of a blocking event from 16 December 1996 to 8 January 1997 are shown in Fig. 6a, and the planetary-scale streamfunction field consisting of waves 1–3 and the synoptic-scale (equal to or greater than wavenumber 8) streamfunction field are presented in Figs. 6b and 6c, respectively.

Figure 6a shows that a blocking event occurs over the North Atlantic during the period 6 December 1996 to 8 January 1997. A weak blocking high ridge appears on 16 December, which is regarded as an incipient blocking and this is then amplified by the synoptic-scale eddies into a blocking circulation that persists for nearly 20 days. It is clear that the blocking region is occupied by several isolated cyclonic and anticyclonic vortices. This blocking flow pattern is basically similar to Fig. 3a, but the detailed structure is slightly different. On the other hand, if and when the synoptic-scale waves (eddies) are filtered out and zonal wavenumbers 1–3 are retained, the basic characteristics of the observed blocking pattern in Fig. 6b are also similar to the idealized model blocking obtained in Fig. 3a, but the detailed structure is still different. This difference is mainly caused by the lack

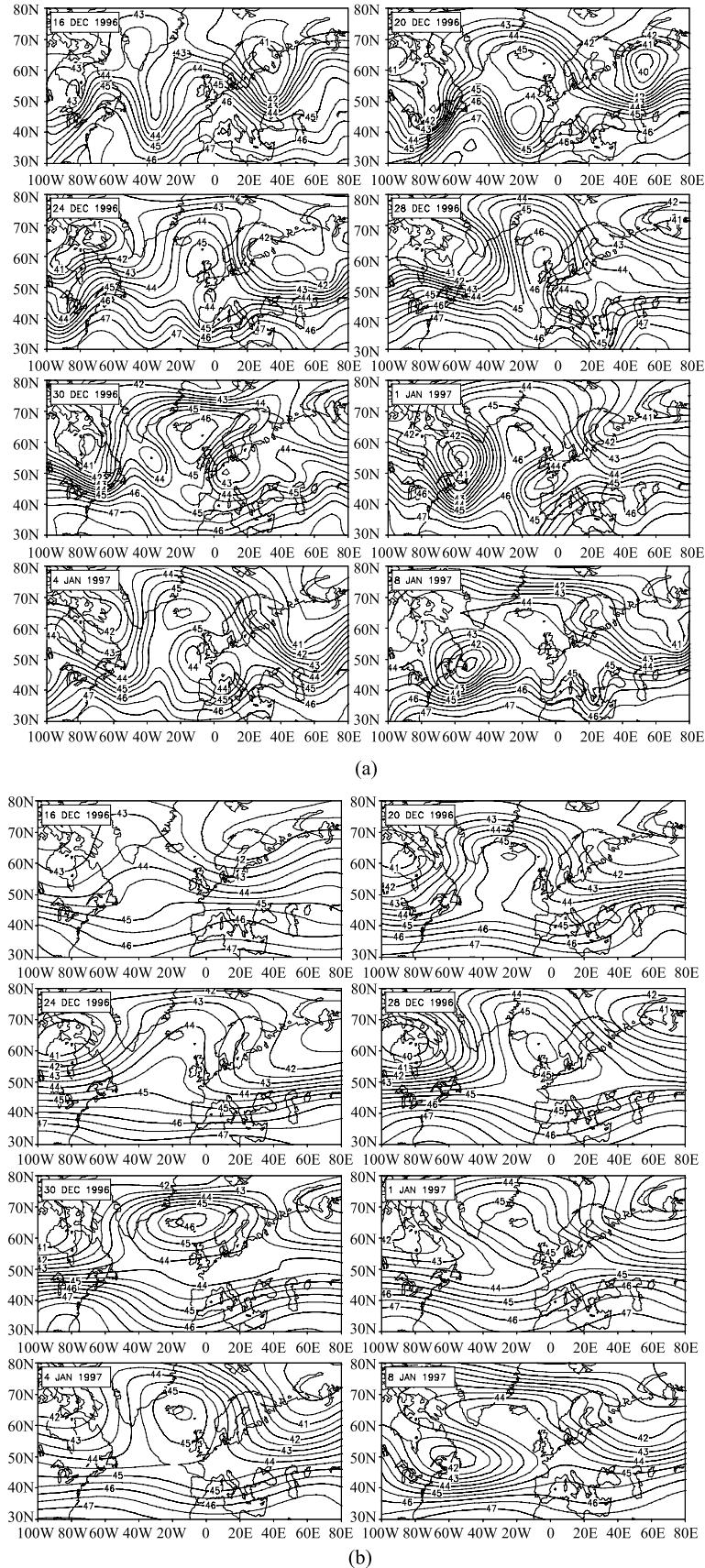
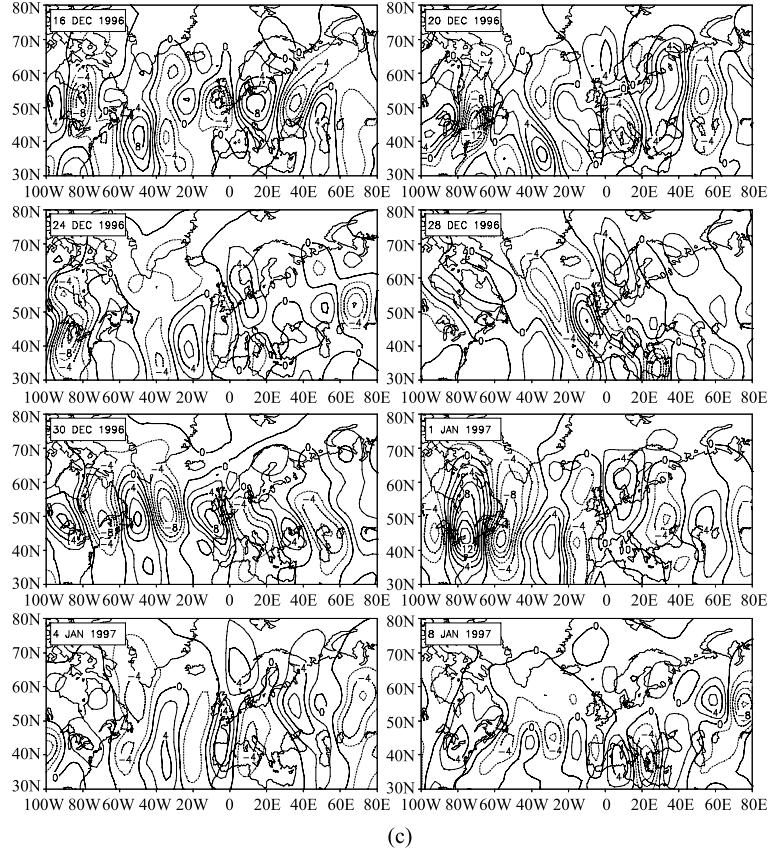


Fig. 6. 500-hPa geostrophic streamfunction of a blocking event over the North Atlantic Ocean during the period from 16 December 1996 to 8 January 1997. (a) Total field, $CI=5 \times 10^6 \text{ m}^2 \text{ s}^{-1}$; (b) Planetary-scale field, $CI=5 \times 10^6 \text{ m}^2 \text{ s}^{-1}$; (c) Synoptic-scale field, $CI=2 \times 10^6 \text{ m}^2 \text{ s}^{-1}$.



(c)

Fig. 6. (Continued).

of orography, diabatic heating, baroclinicity, and Rossby wave packet propagation. These factors should also contribute to blocking events observed in the real atmosphere.

As described by Tung and Lindzen (1979), the blocking ridge was seen as a linear resonance between stationary long waves and external forcing due to large-scale topography and diabatic heating. Nakamura et al. (1997) found that Rossby wave packet propagation appears to play an important role for Atlantic blocks. Because these factors are not involved in our model, the blocking pattern obtained here is naturally unrealistic. This is why there is a large discrepancy between Figs. 3a and 6a. Even so, our model can basically capture the basic characteristics of the life cycle of observed blocking associated with synoptic-scale eddies. Of course, inclusion of the orography, diabatic heating, baroclinicity, and Rossby wave packet propagation will make the model blocking obtained here more realistic. Figure 6c shows the instantaneous streamfunction fields of synoptic-scale waves. We see that during the initial stages of blocking establishment (for example on 16 December) the synoptic-scale waves possess a monopole structure, which is similar to that found in Fig. 3b. The synoptic eddies will be gradu-

ally enhanced and split into two branches around the blocking region with the establishment of blocking. This behavior is particularly noticeable during the period from 18 December to 3 January. In our highly idealized model, the observed behaviour of synoptic-scale eddies can be more or less captured (Fig. 3b). On the other hand, we note that the composite field of the medium-scale waves (zonal wavenumbers 5–7) is almost not influenced by the synoptic-scale waves during the blocking onset (not shown). As demonstrated by Fig. 7b, zonal wavenumber 2 is remarkably amplified by the synoptic-scale waves. This at least indicates that the medium-scale waves are unimportant and do not assist in the planetary-scale amplification of blocking. Thus, the medium-scale waves should be filtered out in order to accentuate the role of synoptic-scale waves in exciting blocking.

Malguzzi (1993) established a link between dipole blocking and synoptic-scale eddies. However, he did not present the instantaneous change of blocking associated with synoptic-scale eddies. In particular, neither how the weak blocking ridge usually observed in the real atmosphere can be evolved into a blocking circulation by synoptic-scale eddies or how the synoptic-scale eddies are modulated by the blocking

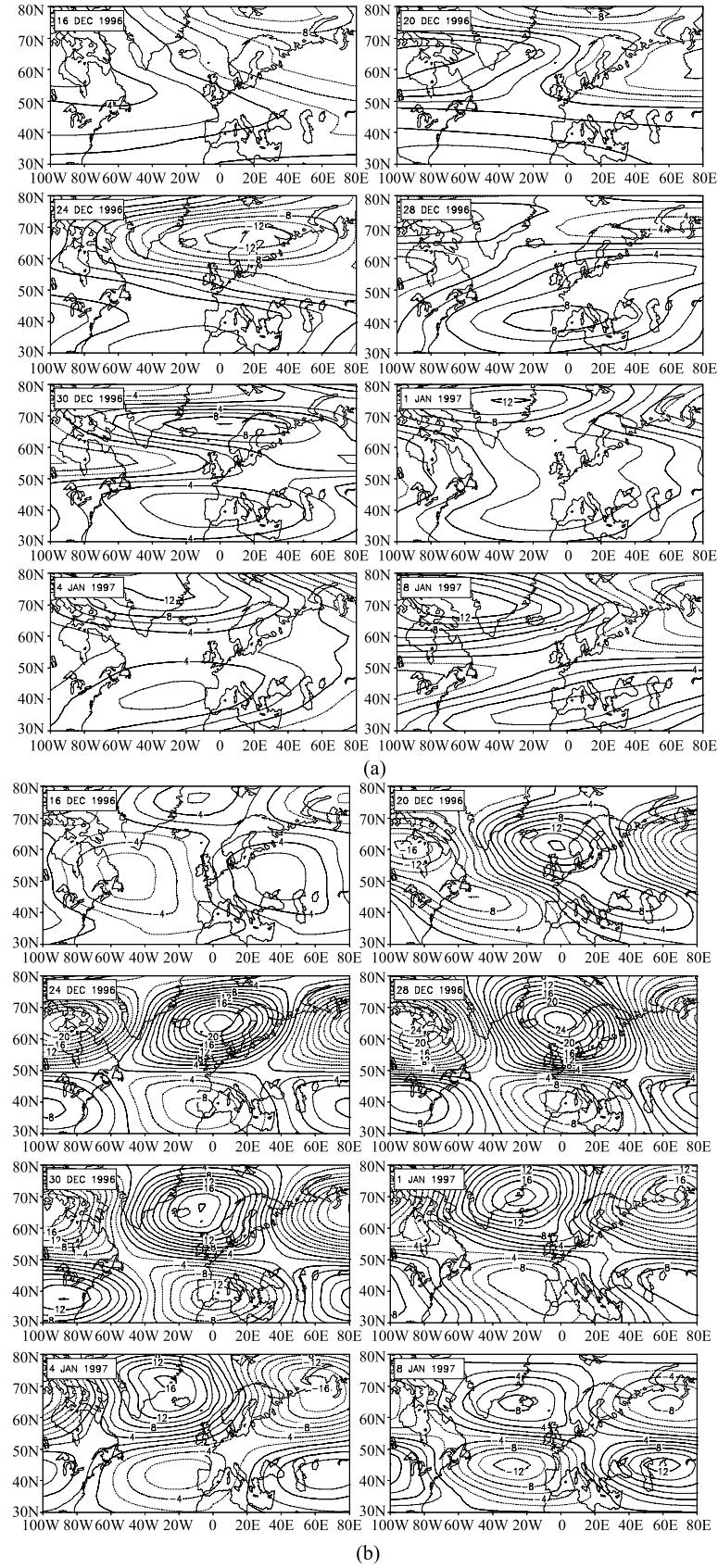


Fig. 7. Instantaneous streamfunction fields of zonal wavenumber 1–3 components for the blocking in Fig. 6. $CI = 2 \times 10^6 \text{ m}^2 \text{ s}^{-1}$: (a) Zonal wavenumber-1 component; (b) Zonal wavenumber-2 component; (c) Zonal wavenumber-3 component.

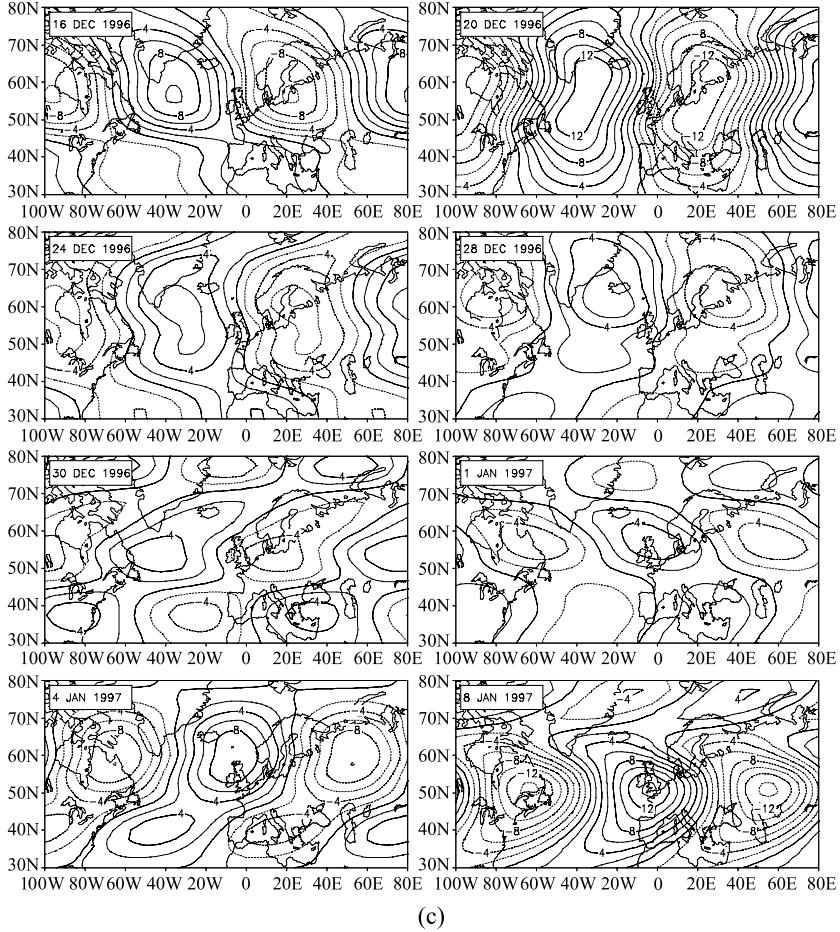


Fig. 7. (Continued).

flow was examined.

Although the observed results presented here can basically confirm the validity of our theoretical model, we should at least provide further evidence that the amplification of wavenumber 2 by synoptic-scale waves is dominant relative to the other two waves. For the real blocking event in Fig. 6a, the instantaneous streamfunction fields of wave components 1, 2, and 3 excluding the wave zero component are shown in Fig. 7a–c, respectively.

It is found in Fig. 7 that both wavenumbers 1 and 3 have a dominant monopole meridional structure throughout the life cycle of blocking, but wavenumber 2 possesses a dipole meridional structure in this blocking event. The comparison with Fig. 3 shows that the three planetary-scale waves observed in Fig. 7 almost have the same meridional structure as the quasi-resonant triad waves considered in our theoretical model. Accordingly, our theoretical blocking model can describe the changes of planetary-scale and synoptic-scale fields in Figs. 6b–c. In order to see the role of synoptic-scale waves in producing a block-

ing circulation more clearly, we will present the time-evolution of the amplitude of each planetary-scale wave for zonal wavenumbers 1–3. In this paper, the mean value of the amplitude of each planetary wave from 45°N to 70°N is seen as the real amplitude of each planetary-scale wave observed because the blocking high mainly occupies the region between 45°N and 70°N.

It is easy to define

$$M_Z(t, n) = \frac{1}{\varphi_2 - \varphi_1} \int_{\varphi_1}^{\varphi_2} A(t, n, \varphi) d\varphi, \quad (15)$$

where $A(t, n, \varphi)$ is the amplitude of the n^{th} Rossby wave obtained from the harmonic wave decomposition of the 500-hPa height field at latitude φ .

For the blocking event presented in Fig. 6, the time-evolution of M_Z for each wave for zonal wavenumbers 1–3 is shown in Fig. 8.

Figure 8 shows that the amplification of wavenumber 2 with a dipole meridional structure is dominant, while the amplitude variation of the other two waves

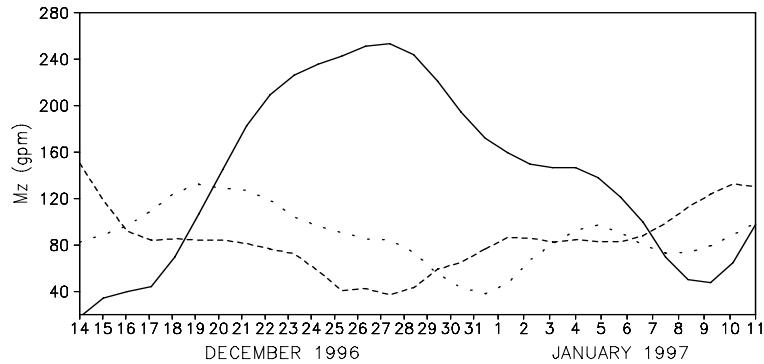


Fig. 8. Time evolution amplitudes, $M_Z(t, n)$, of zonal wavenumbers 1–3 components for the blocking event in Fig. 6. The dashed curve represents the amplitude, $M_Z(t, 1)$, of zonal wavenumber-1 component, the solid curve corresponds to the amplitude, $M_Z(t, 2)$, of zonal wavenumber-2 component, and the point curve represents the amplitude, $M_Z(t, 3)$, of zonal wavenumber-3 component.

is so small that neither wavenumbers 1 and 3 are likely to be influenced by the synoptic-scale waves. This result is consistent with the behavior of quasi-resonant triad Rossby waves obtained in section 3. The amplitude (M_Z) of wavenumber 2 increases from 40 gpm on 16 December to about 260 gpm on 28 December 1996, and then decreases to about 50 gpm by 8 January 1997. The amplitude variation of the other waves is very small compared to that of wavenumber 2. Thus, the amplification of wavenumber 2 dominates the establishment of a blocking circulation. In this process, synoptic-scale waves likely contribute to wavenumber 2 having a dipole meridional structure.

Figure 9 shows the instantaneous distribution of $-J(\psi', \nabla^2 \psi')_P$ at 500-hPa. It is easy to see that the observed $-J(\psi', \nabla^2 \psi')_P$ field possesses a dipole meridional structure and its time-evolution almost exhibits the same change as that of $-J(\psi', \nabla^2 \psi'_1)_P$ presented in Fig. 4. Figure 10 shows the instantaneous field of the planetary-to-medium-scale interaction term denoted by J_{PM} . It is found that the field J_{PM} exhibits a triple monopole meridional structure. This does not force zonal wavenumber 2 with a dipole meridional structure in Fig. 7b even though the field J_{PM} has large amplitude. In fact, the amplification of zonal wavenumber 2 is quasi-resonantly forced by the dipole eddy forcing induced by the synoptic-scale waves. This result is also supported by the diagnostic study of Franzke et al. (2000). Because the dominant zonal wavenumber 2 possesses a dipole structure and the composite field of zonal waves 4–7 has a monopole structure, the field J_{PM} possesses inevitably a triple monopole meridional structure. However, why does the field J_{PM} possess large planetary-scale components during the blocking onset and maintenance? This is easily interpreted. This is because the field J_{PM} con-

tains the amplitude of zonal wavenumber 2. When zonal wavenumber 2 is gradually amplified during the blocking period, the field J_{PM} exhibits an enhancement. Thus, it is inevitable that the field J_{PM} possesses a large planetary-scale component during the blocking period. The field J_{PM} that possesses a large planetary-scale component is actually a result of blocking onset rather than a cause. This trend of the field J_{PM} is easily misunderstood to play an important role for blocking onset. The observational result presented by Hansen and Chen (1982, Fig. 5a) also clearly indicates that the planetary-scale components of the interactions between zonal waves 1–4 and all other waves (5–10) possess the same trend as the growth and decay of blocking waves (1–4). Their diagnostic study confirmed that the interactions between zonal waves 1–4 and all other waves (5–10) that possess enhanced planetary-scale components during the blocking onset are indeed a result of blocking onset rather than a cause. At the same time, we can find from their Fig. 5b that the eddy kinetic energy decreases with the growth of the blocking kinetic energy. This further indicates that the blocking occurs through the upscale transfer of the eddy kinetic energy, while the planetary-to-medium-scale interactions do not assist in the planetary-scale amplification of blocking. Thus, it is natural to exclude the medium-scale waves in Eqs. (3a–b) in order to avoid the contamination of the role of synoptic-scale waves in blocking onset by medium-scale waves.

Further, it is certain that the amplification of zonal wavenumber 2 with a dipole meridional structure may be, to large extent, due to the dipole eddy forcing induced by synoptic-scale waves. As a result, the increase (decrease) of the amplitude of the zonal wavenumber 2 component can basically describe the

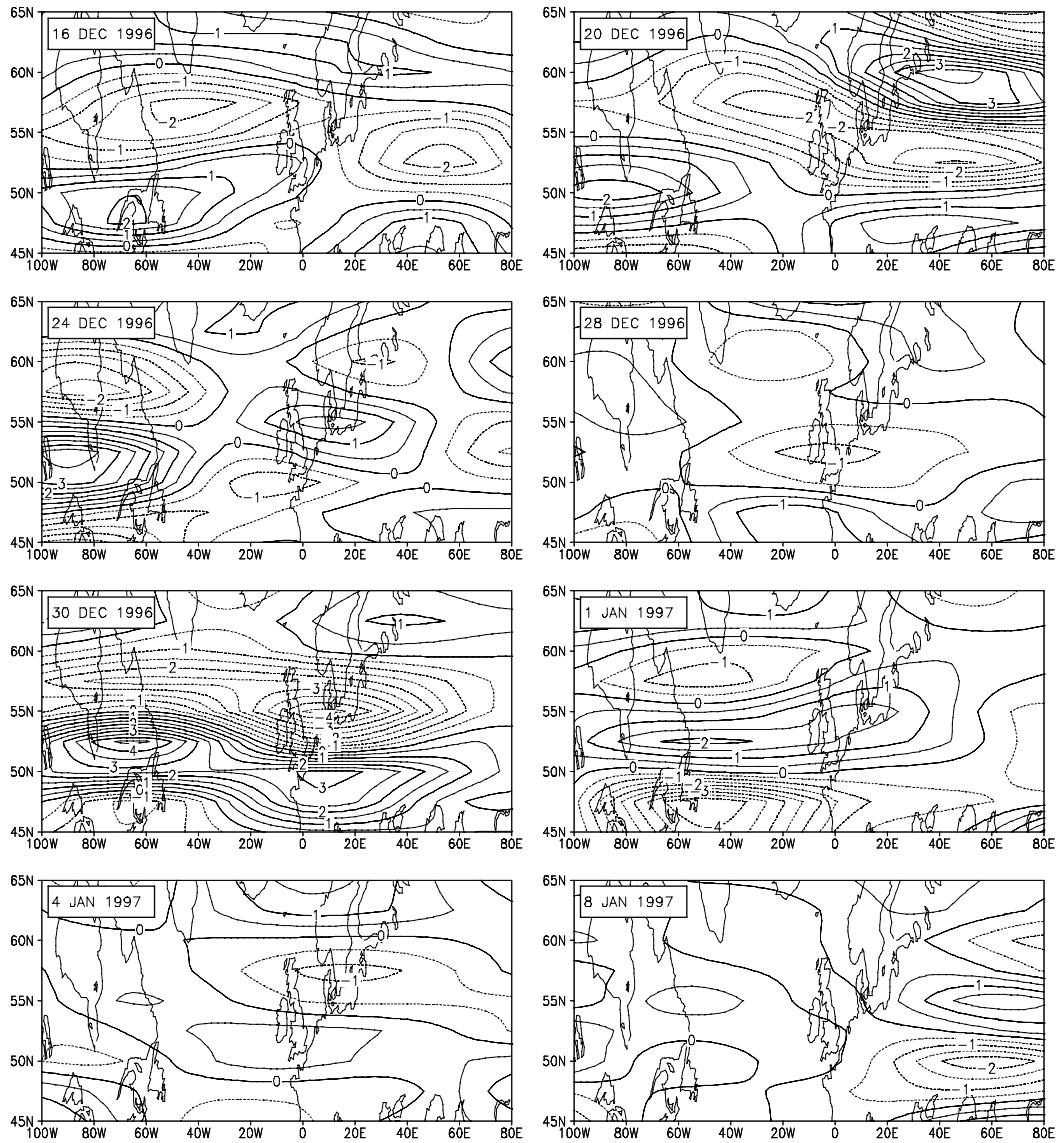


Fig. 9. Instantaneous distribution of $-J(\psi', \nabla^2\psi')_P$ over 500-mb for blocking case depicted in Fig. 6. The solid curve denotes the positive values and the dashed curve represents the negative values. $CI=0.5 \times 10^{-10} \text{ s}^{-2}$.

onset (decay) of a blocking circulation. Thus, in the blocking event presented here, the onset, maintenance, and decay of blocking can crudely be represented by the temporal evolution of wavenumber 2 amplified by synoptic-scale waves in the quasi-resonant triad interaction. This confirms our theory that the occurrence of split-flow blocking is associated with a strong concentration of the wave energy of zonal wavenumber-2 component. The concentration of planetary-scale wave energy is mainly provided by synoptic-scale waves, but is relatively weak only through the quasi-resonant triad interaction. If synoptic-scale waves are not involved, in our model the planetary waves in a quasi-resonant triad interaction cannot achieve the large am-

plitude required for creation of blocking circulation (figures omitted). Thus, the amplification of planetary scale dipole waves by the synoptic-scale waves seems to be very important for the generation of split-flow blocking in the mid-high latitudes. It should be noted that the spherical harmonic decomposition of the planetary-scale field of blocking can tell us which of the planetary-scale waves related to blocking onset is dominant.

Since external forcing such as large-scale topography, diabatic heating, baroclinicity, Rossby wave propagation, etc, are excluded in the model, the theoretical model result does not correspond to real blocking cases observed in the mid-high latitude atmosphere. Al-

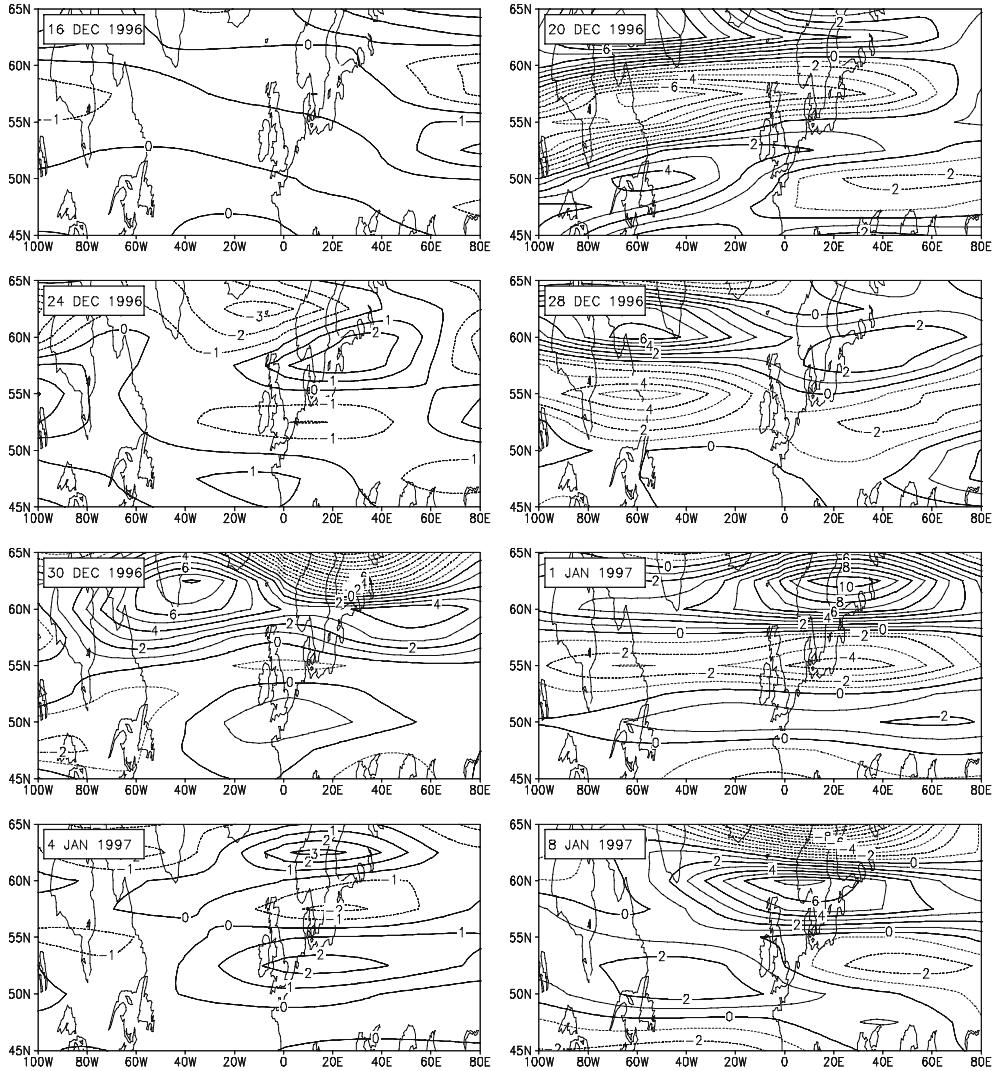


Fig. 10. Instantaneous distribution of $J_{PM} = -J(\psi_{1-3}, \nabla^2 \psi_{4-7})_P - J(\psi_{4-7}, \nabla^2 \psi_{1-3})_P$ over 500-hPa for blocking case depicted in Fig. 6. The solid curve denotes the positive values and the dashed curve represents the negative values. $CI = 0.5 \times 10^{-10} \text{ s}^{-2}$.

though the model does not directly discriminate longitudinally in the location of the split-flow blocking, it indirectly accounts for the dependence of its location on longitude. The main reason is that the model blocking obtained here is attributed to the forcing of synoptic-scale waves that usually occur over the two oceans. However, if the external forcing is excluded, the phase of the blocking flow cannot be determined (Colucci et al., 1981). More recently, the numerical study of Nakamura et al. (1997) indicates that the feedback induced by the synoptic-scale waves accounts for more than 75% of the observed amplification for the Pacific blocking and less than 45% for the European blocking. Although only a blocking case in the Atlantic is used in this paper, we can at least indicate that the mechanism here is at work for some of blocking events.

This suggests in part that the other factors might play a certain role and should be included in our model so that more realistic blocking patterns can be obtained.

6. Conclusion and discussion

In this paper, a quasi-resonant triad interaction model in which the planetary-scale diffluent flow prior to blocking onset is represented by the sum of zonal wavenumbers 1 (monopole), 2 (dipole), and 3 (monopole) and is constructed to investigate the interaction between a planetary-scale diffluent flow and synoptic-scale waves by defining planetary-scale waves as wavenumbers 1–3 and synoptic-scale waves by wavenumbers greater than or equal to 9. This leads to self-interactions among the synoptic-scale

waves projecting onto the planetary-scale while the planetary-scale projection of the interactions between synoptic- and planetary-scale waves vanishes. This formalism is not a mathematical assumption, but is based on the physical considerations. In a quasi-resonant triad interaction model we have investigated the interaction between planetary-scale waves, composed of zonal wavenumbers 1 (monpole), 2 (dipole), and 3 (monopole), and synoptic-scale waves excited by a synoptic-scale vorticity source fixed upstream of an incipient blocking region during the life cycle of blocking. It is shown that if the eddy forcing from synoptic-scale waves is directly imposed on wavenumber 2 having a dipole meridional structure, the wavenumber-2 component is amplified significantly and exhibits a noticeable soliton-oscillation. At the same time, the amplitude variation of the other two planetary waves is so small that they become unimportant in the development of blocking. The superposition of the dominant wavenumber-2 component and the other two planetary waves exhibits the life cycle of blocking, which is consistent with observed blocking. On the other hand, our model can simulate accurately how the planetary-scale circulation changes during the passage of imbedded smaller-scale disturbances and how these smaller-scale perturbations are modulated. Finally, a case study of observed blocking is presented to confirm our theory.

It should be pointed out that our theory is no-

ticeably different from the resonant wave interaction theory proposed by Colucci et al. (1981). In their theory the split-flow configuration is created through the transfer of kinetic energy from two waves with a single meridional structure (monopole) to a third component with a double meridional structure (dipole), but not through the eddy forcing. However, our further investigation here indicates that the energy transfer from two planetary waves having monopole meridional structure to a third planetary wave with a dipole meridional structure is so weak that no blocking can be created. Conversely, the planetary-scale kinetic energy having dipole meridional structure provided by synoptic-scale waves is so strong that a strong split-flow blocking can be created, but the other planetary waves seem to only control the pattern of an incipient blocking.

However, it should be noted that although the flow pattern of the theoretical model blocking obtained here looks like observed blocking, other factors such as medium-scale waves, large-scale topography, diabatic heating, baroclinicity, Rossby wave propagation, etc. are not involved in our model. It is also unclear the mechanism proposed here is at work for all blocking events. These aspects need further investigation.

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APPENDIX: Coefficients of Solution (13)

The coefficients of Eq. (13) are defined as

$$\begin{aligned}
 \Re_i &= \frac{Q_i}{(\beta + F\bar{u})(k_1 + \tilde{k}_i) - [\bar{u}(k_1 + \tilde{k}_i) - (\tilde{\omega}_i + \omega_1)][(k_1 + \tilde{k}_i)^2 + 4m^2 + F]}, \\
 \rho_i &= \frac{Q_i}{(\beta + F\bar{u})(-k_1 + \tilde{k}_i) - [\bar{u}(-k_1 + \tilde{k}_i) - (\tilde{\omega}_i - \omega_1)][(-k_1 + \tilde{k}_i)^2 + 4m^2 + F]}, \\
 J_{ij} &= \frac{H_i}{(\beta + F\bar{u})(\tilde{k}_i + k_2) - [\bar{u}(k_2 + \tilde{k}_i) - (\tilde{\omega}_i + \omega_2)]\{(k_2 + \tilde{k}_i)^2 + [2 - (-1)^j]^2 m^2 + F\}}, \\
 S_{ij} &= \frac{H_i}{(\beta + F\bar{u})(\tilde{k}_i - k_2) - [\bar{u}(\tilde{k}_i - k_2) - (\tilde{\omega}_i - \omega_2)]\{(\tilde{k}_i - k_2)^2 + [2 - (-1)^j]^2 m^2 + F\}}, \\
 \chi_i &= \frac{E_i}{(\beta + F\bar{u})(\tilde{k}_i + k_3) - [\bar{u}(\tilde{k}_i + k_3) - (\tilde{\omega}_i + \omega_3)][(\tilde{k}_i + k_3)^2 + 4m^2 + F]}, \\
 \gamma_i &= \frac{E_i}{(\beta + F\bar{u})(\tilde{k}_i - k_3) - [\bar{u}(\tilde{k}_i - k_3) - (\tilde{\omega}_i - \omega_3)][(\tilde{k}_i - k_3)^2 + 4m^2 + F]} \\
 \sigma &= \frac{(\tilde{k}_2 - \tilde{k}_1)^2(\tilde{k}_1 + \tilde{k}_2)}{(\beta + F\bar{u})(\tilde{k}_1 + \tilde{k}_2) - [\bar{u}(\tilde{k}_1 + \tilde{k}_2) - (\tilde{\omega}_1 + \tilde{\omega}_2)][(\tilde{k}_1 + \tilde{k}_2)^2 + 4m^2 + F]}, \\
 Q_i &= k_1^2 + m^2 - (\tilde{k}_i^2 + m^2), \\
 H_i &= k_2^2 + 4m^2 - (\tilde{k}_i^2 + m^2), \\
 E_i &= k_3^2 + m^2 - (\tilde{k}_i^2 + m^2) \quad (i = 1, 2; j = 1, 2).
 \end{aligned}$$

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